Which option would you choose?
① \$5 million ② \$ doubled each day for a whole month (beginning with 1¢)

The "Magic Penny" Problem

Suppose that a Wizard gives you a "Magic Penny" that doubles in value each day. If the Wizard gives you the "Magic Penny" on the first of the month,

A) What will be its value (in dollars) at the end of the week (7^{th})?

See calender : 64¢

B) What will be its value (in dollars) at the end of the 2^{nd} week?

See calender :

C) What will be its value (in dollars) the end of the month?
(hint: It's easier to generalize a formula than to fill out the calendar)

Let x be # of days

$\therefore \quad Y = 2^{x-1}$

$Y = 2^{31-1} = 2^{30} = 1073741824$

$\therefore$ \$10,737,418.24

SWEET DEAL! \$10million baby

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
1 1¢	2 2¢	3 4¢	4 8¢	5 16¢	6 32¢	7 64¢
8 \$1.28	9 \$2.56	10 \$5.12	11 \$10.24	12 \$20.48	13 \$40.96	14 \$81.92
15 ...	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31 ?	1	2	3	4

Honors Advanced Algebra **DATE:**________________

7.1 Exploring Exponential Models

Target 5A. Graph, transform and identify the key features of the graph of an exponential and logarithmic function

Motivation

Determine a formula for the exponential function whose values are given. Use the model to predict the population (in millions) for 2010.

Year	1900	1910	1920	1930	1940	1950	1960	1970	1980	1990	2000
	0	1	2	3	4	5	6	7	8	9	10
Population (in millions)	76.2	92.2	106.0	123.2	132.2	151.3	179.3	203.3	226.5	248.7	281.4

$y = 80.551(1.137)^x$ Regression model [See attachment]

Since 2000 corresponds to 10, 2010 corresponds to 11. Therefore the population (in millions) for 2010 is: $y = 80.551(1.137)^{11} \approx 330.701$ million

Exponential Function	**Exponential Growth**		**Exponential Decay**	
	Conditions	*Example*	*Conditions*	*Example*
$f(x) = a \cdot b^x$	$a > 0$ $b > 1$	$f(x) = 2(3)^x$ $g(x) = 0.5(1.12)^x$ $h(x) = 2^x$	$a > 0$ $0 < b < 1$	$h(x) = (0.5)^x$ $g(x) = 0.25(0.99)^x$ $p(x) = \frac{1}{4}\left(\frac{1}{10}\right)^x$

Graphing Exponential Functions

1. Sketch the graph of $y = 3^x$. Identify each function as an example of exponential growth or decay. Then state the function's domain, range, y-intercept, and asymptote.

x	y
−4	0.01
−2	0.1
−1	0.3
0	1
1	3
2	9

→ Used Nspire to generate Table

$\lim_{x \to \infty} f(x) = \infty$

$\lim_{x \to -\infty} f(x) = 0$

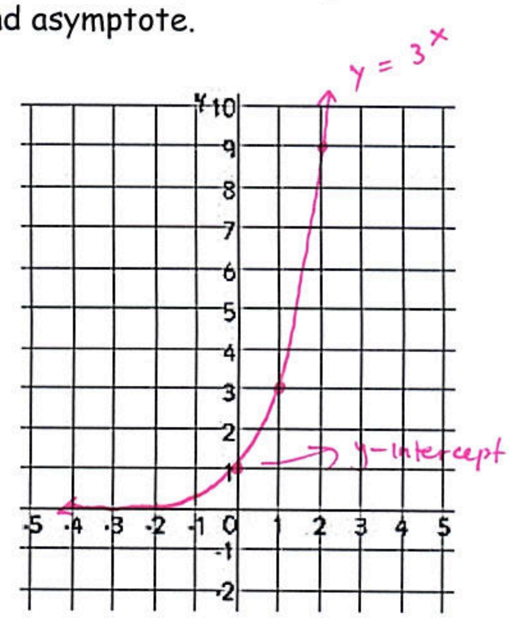
① $y = 3^x$ is exponential Growth b/c $b = 3 > 1$

② Domain: $(-\infty, \infty) = \mathbb{R}$

③ Range: $(0, \infty)$

④ y-intercept @ $(0, 1)$

⑤ H.A. @ $y = 0$ ↓ Horizontal Asymptote



2. Sketch the graph of $y = 2\left(\frac{1}{4}\right)^x$. Identify each function as an example of exponential growth or decay. Then state the function's domain, range, y-intercept, and asymptote.

x	y
−1	8
0	2
1	0.5
2	0.125
3	0.03125

→ Generated table using Nspire

① $y = 2\left(\frac{1}{4}\right)^x$ is exponential Decay b/c $b = \frac{1}{4} < 1$

② Domain: $(-\infty, \infty) = \mathbb{R}$

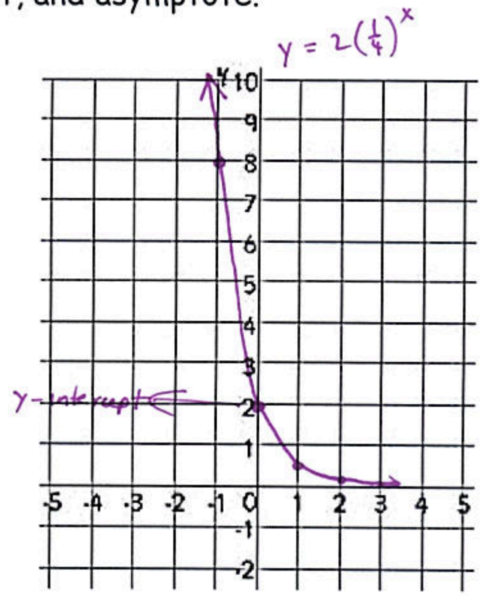
③ Range: $(0, \infty)$

④ y-intercept @ $(0, 2)$

⑤ H.A. @ $y = 0$

$\lim_{x \to \infty} f(x) = 0$

$\lim_{x \to -\infty} f(x) = \infty$



In general, an equation of the form $y = ab^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$, is called an *exponential function* with base *b*.

Determine whether each function represents growth or decay without using a calculator.

3. $y = \left(\frac{1}{5}\right)^x$
Decay b/c $b = \frac{1}{5} < 1$

4. $y = 3(4)^x$
Growth b/c $b = 4 > 1$

5. $y = 7(1.2)^x$
Growth b/c $b = 1.2 > 1$

6. $y = (0.7)^x$
Decay b/c $b = 0.7 < 1$

7. $y = \frac{1}{2}(3)^x$
Growth b/c $b = 3 > 1$

8. $y = 10\left(\frac{4}{3}\right)^x$
Growth b/c $b = \frac{4}{3} > 1$

Property of Equality for Exponential Functions: If *b* is a positive number other than 1, then $b^x = b^y$ if and only if $x = y$.
Example: If $\boxed{2}^x = \boxed{2}^8$, then $x =$ <u>8</u>.

Solve Exponential Equations

9. $3^{2n+1} = 81$

$\boxed{3}^{2n+1} = \boxed{3}^4$

$2n + 1 = 4$
$-1 \quad -1$
$2n = 3$
$\boxed{n = \frac{3}{2}}$

10. $4^{2x} = 8^{x-1}$

$(2^2)^{2x} = (2^3)^{x-1}$
$2^{2 \cdot 2x} = 2^{3(x-1)}$
$\boxed{2}^{4x} = \boxed{2}^{3(x-1)}$
$4x = 3(x-1)$
$4x = 3x - 3 \Rightarrow \boxed{x = -3}$

11. $4^{9n-2} = 256$

$(2^2)^{9n-2} = 2^8$
$\boxed{2}^{2(9n-2)} = \boxed{2}^8$
$\frac{2(9n-2)}{2} = \frac{8}{2}$
$9n - 2 = 4$
$+2 \quad +2$
$\Rightarrow \frac{9n}{9} = \frac{6}{9} \Rightarrow n = \frac{6}{9} = \boxed{\frac{3}{2}}$

12. $3^{5x} = 9^{2x-1}$

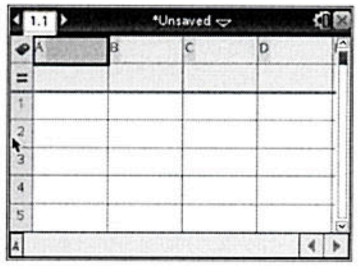
$3^{5x} = (3^2)^{2x-1}$
$\boxed{3}^{5x} = \boxed{3}^{2(2x-1)}$
$5x = 2(2x-1)$
$5x = 4x - 2$
$\boxed{x = -2}$

How about
① $y = 0.5^{-x}$?
② $y = 2^{-x}$?

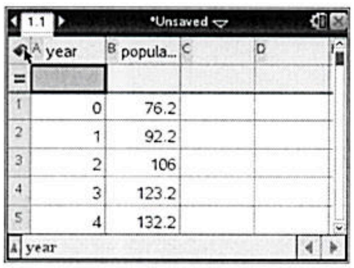
① $y = 0.5^{-x}$
$= \left(\frac{1}{2}\right)^{-x} = \frac{1^{-x}}{2^{-x}}$
$= \frac{2^x}{1^x} = \left(\frac{2}{1}\right)^x = 2^x$
So Growth :)

② $y = 2^{-x}$
$= \frac{1}{2^x}$
$= \frac{1^x}{2^x} = \left(\frac{1}{2}\right)^x$
So Decay :(

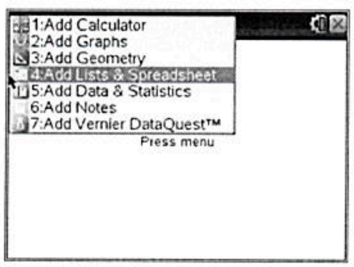
Step 1: Go to Home and Add a List & Spreadsheet Document

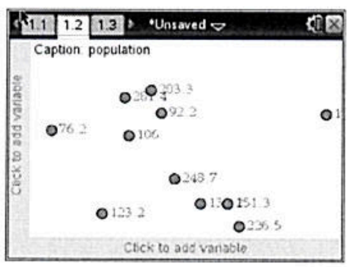


Step 2: Enter "year" in cell A and "population" in cell B. Then enter all the data. Begin with 0 for year 1900, 1 for year 1910, 2 for year 1920, etc.

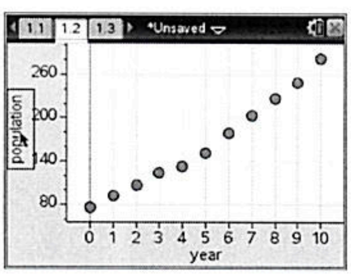


Step 3: Once your data has been entered, press ctrl and doc. Then Add Data & Statistics





Step 4: Click to add variable and choose "years". Then click to add variable and choose "population"



Step 5: Finally go to menu, analyze, regression, and show exponential