7-3 & 7-6 Logarithmic Functions as Inverses

In general, the inverse of $y = b^x$ is $x = b^y$. In $x = b^y$, y is called the **logarithm**, base b, of x. Usually written as $y = \log_b x$ and is read "y equals log base b of x".

Logarithm with base b: Suppose $b > 0$ and $b \neq 0$. For $x > 0$, there is a number y such that $\log_b x = y$ if and only if $b^y = x$.

Exponential Form	Logarithmic Form
$b^y = x$ "If you don't know what to do, do the canoe"	$\log_b x = y$ (but $b \neq 1$ and $b > 0$)

Common Log → has base __10__

$\log_{10} x = \log x$

Natural Log → has base __$e \approx 2.718\ldots$__

$\log_e x = \ln x$

Examples

Using a calculator, evaluate the logarithmic expression. "Round to 3 decimal places"

1. $\log 4$ 0.602

2. $\ln 2$ 0.693

3. $\log_2 5$ 2.322

Properties of Logs/Natural Logs b/c = because

$\log_b 1 =$ __0__ b/c $b^{0} = 1$ . . . $e^{0} = 1$ b/c $\ln 1 =$ __0__

$\log_b b =$ __1__ b/c $b^{1} = b$ . . . $e^{1} = e$ b/c $\ln e =$ __1__

$\cancel{\log_b b}^y =$ __y__ b/c $b^{y} = b^y$ . . . $e^{y} = e^y$ b/c $\cancel{\ln e}^y =$ __y__

$\cancel{b}^{\log_b y} =$ __y__ b/c $\log_b y = \log_b y$ . . . $\ln y = \ln y$ b/c $\cancel{e}^{\ln y} =$ __y__

"Do the Canoe" ... if you don't know what to do... ☺

Logarithmic to Exponential Form

Write each equation in exponential form.

1. $\log_8 1 = 0$ $\Leftrightarrow$ $8^0 = 1$

2. $\log_2 \frac{1}{16} = -4$ $\Leftrightarrow$ $2^{-4} = \frac{1}{16}$

3. $\log_3 9 = 2$ $\Leftrightarrow$ $3^2 = 9$

4. $\log \frac{1}{100} = -2$ $\Leftrightarrow$ $\log_{10} \frac{1}{100} = -2$ $\Leftrightarrow$ $10^{-2} = \frac{1}{100}$

Exponential to Logarithmic Form

Write each equation in logarithmic form.

5. $10^3 = 1000$ $\Leftrightarrow$ $\log_{10} 1000 = 3$ $\Leftrightarrow$ $\log 1000 = 3$

6. $9^{\frac{1}{2}} = 3$ $\Leftrightarrow$ $\log_9 3 = \frac{1}{2}$

7. $5^3 = 125$ $\Leftrightarrow$ $\log_5 125 = 3$

8. $27^{\frac{1}{3}} = 3$ $\Leftrightarrow$ $\log_{27} 3 = \frac{1}{3}$

Evaluate Logarithmic Expressions

9. $\log_2 64$

Let $\log_2 64 = r$

Then, $2^r = 64$

$2^r = 2^6$

$\therefore r = 6$

So, $\boxed{\log_2 64 = 6}$

10. $\log_3 \frac{1}{27}$

Let $\log_3 \frac{1}{27} = r$

Then, $3^r = \frac{1}{27}$

$3^r = \frac{1}{3^3}$

$3^r = 3^{-3}$

$\therefore r = -3$

So, $\boxed{\log_3 \frac{1}{27} = -3}$

11. $\log_2 \sqrt{8}$

Let $\log_2 \sqrt{8} = r$

Then, $2^r = \sqrt[2]{8}$

$2^r = 8^{\frac{1}{2}}$

$2^r = (2^3)^{\frac{1}{2}}$

$2^r = 2^{\frac{3}{2}}$

$\therefore r = \frac{3}{2} = 1.5$

So, $\boxed{\log_2 \sqrt{8} = \frac{3}{2}}$

Inverse Property of Exponents and Logarithms

Evaluate each expression.

12. $\log_6 6^8$ $= 8$

13. $\log_9 9^2$ $= 2$

14. $3^{\log_3(4x-1)}$ $= 4x - 1$

15. $7^{\log_7 21}$ $= 21$

16. $\log_5 5^{x-1}$ $= x - 1$

17. $9^{\log_9 37}$ $= 37$

Logarithmic Equation: an equation that contains one or more logarithms.

Solving Logarithmic Equations

Solve each logarithmic equation.

18. $\log_4 n = \frac{5}{2}$

$4^{5/2} = n$

$(2^2)^{5/2} = n$

$2^{\frac{10}{2}} = n$

$2^5 = n$

19. $\log_8 n = \frac{4}{3}$

$8^{\frac{4}{3}} = n$

$(2^3)^{\frac{4}{3}} = n$

$2^4 = n$

$\boxed{16 = n}$

20. $\log_9 x = \frac{3}{2}$

$9^{3/2} = n$

$(3^2)^{3/2} = n$

$3^3 = n$

$\boxed{27 = n}$

21. $\log_{\frac{1}{10}} x = -3$

$\left(\frac{1}{10}\right)^{-3} = x$

$\frac{1^{-3}}{10^{-3}} = x$

$\frac{10^3}{1^3} = x$

$\frac{1000}{1} = x$

$\boxed{1000 = x}$

Property of Equality for Logarithmic Functions:
If b is a positive number other than 1, then $\log_b x = \log_b y$ if and only if $x = y$

Solving Equations with Logarithms on Each Side

Solve each logarithmic equation.

22. $\cancel{\log_5}(3x - 2) = \cancel{\log_5} x$

$3x - 2 = x$

$-3x \quad -3x$

$-2 = -2x$

$\boxed{1 = x}$ ✓

23. $\cancel{\log_7}(8x + 20) = \cancel{\log_7}(x + 6)$

$8x + \cancel{20} = \cancel{x} + 6$

$-x \; -\cancel{20} \quad -\cancel{x} \; -20$

$7x = -14$

$\boxed{x = -2}$ ✓

24. $\cancel{\log_5}(x^2 - 2) = \cancel{\log_5} x$

$x^2 - 2 = x$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x - 2 = 0$ or $x + 1 = 0$

$x = 2 \qquad x = -1$

CHECK (for extraneous solutions):

① $\log_5(x^2 - 2) = \log_5 x$

$\log_5(2^2 - 2) = \log_5 2$

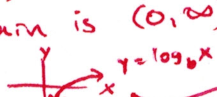
$\log_5 2 = \log_5 2$ ✓

② $\log_5((-1)^2 - 2) = \log_5 -1$

$\log_5 -1 = \log_5 -1$ ✗

$\log_5 -1$ is non-real calculator

Thus, undefined since domain is $(0, \infty)$.



25. $\cancel{\log_3}(x^2) = \cancel{\log_3}(9)$

$x^2 = 9$ "Take square root of both sides"

$x = \pm 3$

CHECK: ① $\log_3(x^2) = \log_3 9$

$\log_3 3^2 = \log_3 9$

$\log_3 9 = \log_3 9$ ✓

② $\log_3(-3)^2 = \log_3 9$

$\log_3 9 = \log_3 9$ ✓

$\therefore$ $x = \pm 3$ are both solutions