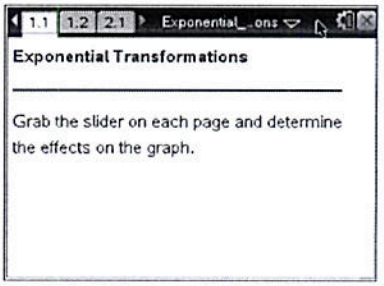
Exponential Transformations

Student Activity

Name _____

Class _____

Open the TI-Nspire document *Exponential_Transformations.tns*.



The graph of any function can be moved on the *x*- and *y*-axes by following a few rules. In this activity, you will discover these rules for exponential functions.

$y = a \cdot \text{base}^{b(x-h)} + k$

Move to page 1.2.

For this activity, the function used is $y = a \cdot 3^{b(x-h)} + k$. This activity's investigations also work for any base b such that $b > 0$ and $b \neq 1$.

1. a. What effect does dragging the *k*-value have on the parent function $y = 3^x$? What happens algebraically to the point (0, 1) in terms of *k* as the graph is translated up or down?

Increasing k-values ⇒ vertical translation up Decreasing k-values ⇒ vertical translation down

   b. Name the transformation, including its distance and direction, when the function $y = 3^x$ changes to $y = 3^x + 2$. How does the point (0, 1) change?

Translation up 2 units. 2 is added to y-coordinate ⇒ (0, 1+2) = (0, 3)

Move to page 2.1.

2. Change the *h*-value by grabbing and dragging the slider.
   a. What happens to the equation and graph when $h > 0$?

Equation changes to exponent x−h. Increasing h-values ⇒ horizontal translation right. Decreasing h values ⇒ horizontal translation left

   b. Christina says that the point (0, 1) on the parent function translates to (–2, 1) when she drags the *h*-value to –2 because the *y*-value is being multiplied by –2. Is her explanation mathematically correct? Explain. Change the *h*-value and confirm your explanation by grabbing and dragging the slider.

No, has nothing to do with multiplication!

The point (0,1) has been translated to the left to (−2,1).

   c. Name the transformation, including its distance and direction, when the function $y = 3^x$ changes to $y = 3^{x-2}$.

Horizontal translation right 2 units.

Move to page 3.1.

3. Change the *a*-value by clicking on the arrows.
   a. When the *a*-value is 0.5, explain why the point (1, 3) becomes the transformed point (1, 1.5).

$y = 0.5(3)^x$
$= 0.5(3)^1$
$= 0.5(3)$
$= 1.5$

So pt (1,3) becomes (1, 1.5)

$a > 1$ or $a < -1$ ⇒ vertical stretch, negative value of "a" ⇒ reflect over x-axis
$-1 < a < 1$, $a \neq 0$ ⇒ vertical shrink (compression)

   b. What happens to the point (1, 3) when the function changes from $y = 3^x$ to $y = 2 \cdot 3^x$? What transformation occurred?

$y = 2 \cdot 3^1 = 2 \cdot 3 = 6$ ⇒ (1, 6). So pt. (1, 3) becomes (1, 6).

The graph becomes vertically stretched by factor of 2.

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4. Change the **b**-value by clicking on the arrows.
   a. When **b** < 0, what happens to the point (0, 1)? If $b < 0$, what happens to the graph?
   Nothing. Negative b-values reflect graph over y-axis
   b. What other effects does the *b*-value have on the graph?
   $b > 1$ or $b < -1$ ⇒ horizontal shrink (compression)
   $-1 < b < 1$; $b \neq 0$ ⇒ horizontal stretch
   c. Suppose the function changes from $y = 3^x$ to $y = 3^{2x}$. Describe the transformation that occurs. Horizontal shrink

Move to page 5.1.

5. Apply what you have learned and change the values of *h* and *k* (by dragging their sliders) and of *a* and *b* (by clicking their arrows) so that in the displayed domain, the solid graph is transformed to the dashed graph. It will say *Correct!* when you have done it correctly.

   Write the function you arrived at here. Describe your thought process of getting to the answer. Are other solutions possible?

   $y = 2 \cdot 3^{0.5(x-3)} + 2$ (must try)

6. David says that positive *a*-values greater than 1 cause the function to stretch vertically. Is he correct? Explain. No. He'll be correct for some values of a, positive values of a; example, from 3 to 5. But if he uses value from 5 to 3, then we have a shrink.

7. Leon says that changing $y = 3^x$ to $y = 3^{x+4}$ results in its graph having a horizontal translation of 4 units to the right. Is Leon correct? Why or why not?

   No, 4 units to the left

   $y = \text{base}^{x-h}$ ⇒ shift right h units
   $y = \text{base}^{x+h}$ ⇒ shift left h units

8. a. Write the function that transforms $y = \sqrt{x}$ horizontally to the left 5 units and has a vertical dilation factor of 3.

   $y = 3\sqrt{x+5}$

   b. Write the function that transforms $y = |x|$ with a vertical translation up 3 units.

   $y = |x| + 3$

Wrap up

9. Describe how a, b, h, and k transforms the graph of $y = a \cdot \text{base}^{b(x-h)} + k$. In summary,

   k : translation up and down
   h : translation left and right (see # 7)
   a : See # 3a
   b : See # 4b