Honors Advanced Algebra **DATE:**________________

Geometric Sequences & Series

Target 5F. Solve problems using the formula for the sum of a finite geometric series.

Geometric Sequence

- You build a geometric sequence by multiplying each term by a constant
- In a geometric sequence, the ratio of any term to its preceding term is a constant value

A **geometric sequence** with a starting value a and a **common ratio** r is a sequence of the form

$a, ar, ar^2, ar^3, \ldots$

1st term → 2nd term → 3rd term → 4th term and so on...

A recursive definition for the sequence has two parts:

$a_1 = a$	initial condition
$a_n = a_{n-1} \cdot r$, for $n > 1$	recursive formula

CHECK: $a_1 = a$ ✓

$a_2 = a_{2-1}r = a_1 r = ar$ ✓✓

$a_3 = a_{3-1}r = a_2 r = ar \cdot r = ar^2$

$a_4 = a_{4-1}r = a_3 r$

$= ar^2 \cdot r$

$= ar^3$ ☺

An explicit definition for this sequence is a single formula:

$a_n = a_1 \cdot r^{n-1}$, for $n \geq 1$

Identifying Geometric Sequences

Is the sequence geometric? If it is, what are a_1 and r?

1. $3, 6, 12, 24, 48, \ldots$

$\frac{6}{3} = \frac{12}{6} = \frac{24}{12} = \frac{48}{24} = \boxed{2}$

- Common ratio is $\boxed{2}$.
- Sequence is Geometric ☺
- $a_1 = 3$
- $r = 2$

2. $3, 6, 9, 12, 15, \ldots$

$2 = \frac{6}{3} \neq \frac{9}{6} = \frac{3}{2}$

The ratios are different.
With no common ratio, the sequence is not geometric

3. $3^5, 3^{10}, 3^{15}, 3^{20}, \ldots$

$\frac{3^{10}}{3^5} = \frac{3^{15}}{3^{10}} = \frac{3^{20}}{3^{15}} = \boxed{3^5}$

- Common ratio is $\boxed{3^5}$.
- The sequence is geometric ☺
- $a_1 = 3^5$
- $r = 3^5$

Analyzing Geometric Sequences

What are the indicated terms of the geometric sequence?

4. The 10th term of the geometric sequence 4, 12, 36, …

$\frac{12}{4} = \frac{36}{12} = \boxed{3}$

1st term: $a_1 = 4$

Common ratio: $r = 3$

$a_n = a_1 \cdot r^{n-1}$

$a_{10} = 4 \cdot 3^{10-1}$

$= 4 \cdot 3^9$

$= 78{,}732$

$\boxed{\therefore\ 10^{th} \text{ term: } a_{10} = 78{,}732}$

5. The second and third terms of the geometric sequence 2, ___, ___, -54, …

1st term: $a_1 = 2$

4th term: $a_4 = -54$

$a_n = a_1 \cdot r^{n-1}$

$a_4 = a_1 \cdot r^{4-1}$

$\frac{-54}{2} = \frac{2 \cdot r^3}{2}$

$-27 = r^3$ Take cube root of both sides.

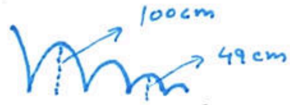
$\therefore r = -3$, the common ratio

Now, begin with 2 and multiply by -3.

$2, -6, 18, -54, \ldots$

$\boxed{\therefore\ 2^{nd} \text{ term: } -6 \quad 3^{rd} \text{ term: } 18}$

Using a Geometric Sequences



6. When a ball bounces, the heights of consecutive bounces form a geometric sequence. What are the heights of the 4th and 5th bounce of a ball that has the geometric sequence 100 cm, ______, 49 cm, 34.3cm, 24cm, ...

you find it

$a_1 = 100$

$a_3 = 49$

$a_n = a_1 r^{n-1}$

$a_3 = a_1 r^{n-1}$

$49 = 100 \cdot r^{3-1}$

$49 = 100 \cdot r^2$

$\frac{49}{100} = r^2$ Take square root of both sides

$\pm \frac{7}{10} = r$

$\boxed{\frac{7}{10} = r}$ r must be positive

Why? Bounces above floor

Portion of height of a bounce that next bounce will reach

$a_4 = ?$ $a_5 = ?$ Use $a_n = a_{n-1} \cdot r$

$a_4 = a_{4-1} \cdot r = a_3 \cdot r = 49 \cdot \frac{7}{10} = 34.3$

$a_5 = a_{5-1} \cdot r = a_4 \cdot r = 34.3 \cdot \frac{7}{10} \approx 24$

$\therefore$ The heights are 34.3 cm and 24 cm.

Geometric Series

A **geometric series** is the sum of the terms of a geometric sequence.

The sum S_n of a finite geometric series $a_1 + a_1 r + a_1 r^2 + \cdots + a_1 r^{n-1}$, $r \neq 1$, is

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

where a_1 is the first term, r is the common ratio, and n is the number of terms.

Finding the Sums of Finite Geometric Series

What is the sum of the finite geometric series?

7. $3 + 6 + 12 + 24 + \cdots + 3072$

$\frac{6}{3} = \frac{12}{6} = \frac{24}{12} = \boxed{2}$ $\therefore r = 2$

1st term: $a_1 = 3$

nth term: $a_n = 3072$

$a_n = a_1 r^{n-1}$

$\frac{3072}{3} = \frac{3 \cdot 2^{n-1}}{3}$

$1024 = 2^{n-1}$

$2^{10} = 2^{n-1}$

$10 = n - 1$

$\boxed{11 = n}$

$S_n = \frac{a_1(1 - r^n)}{1 - r} \Rightarrow S_{11} = \frac{3(1 - 2^{11})}{1 - 2}$

$= 6141$

$\therefore$ Sum of finite geometric series is 6141.

8. $-15 + 30 - 60 + 120 - 240 + 480$

$\frac{30}{-15} = \frac{-60}{30} = \frac{120}{-60} = \frac{-240}{120} = \frac{480}{-240} = \boxed{-2}$

$a_1 = -15$

There are 6 terms:

$S_6 = \frac{-15(1 - (-2)^6)}{1 - (-2)}$

$= \frac{-15(1 - 64)}{3}$

$= \frac{-15(-63)}{3} = -5(-63) = \boxed{315}$