DATE: __________

3.4 Properties of Logarithmic Functions (Target 3C/3E)

Review of Prior Concepts (ACT Warmup)

1. For positive real numbers x, y, and z, which of the following expressions is equivalent to $x^{\frac{1}{2}}y^{\frac{2}{3}}z^{\frac{5}{6}}$?

A. $\sqrt[3]{xy^2z^3}$
B. $\sqrt[6]{xy^2z^5}$
C. $\sqrt[6]{x^3y^2z^5}$
D. $\boxed{\text{D.}}$ $\sqrt[6]{x^3y^4z^5}$
E. $\sqrt[11]{xy^2z^5}$

$x^{1/2} \cdot y^{2/3} \cdot z^{5/6}$

$x^{3/6} \cdot y^{4/6} \cdot z^{5/6}$

$(x^3 y^4 z^5)^{1/6}$

$\sqrt[6]{x^3 y^4 z^5}$

2. In the real numbers, what is the solution of the equation $8^{2x+1} = 4^{1-x}$?

A. $-\frac{1}{3}$
B. $-\frac{1}{4}$
C. $\boxed{\text{C.}}$ $-\frac{1}{8}$
D. 0
E. $\frac{1}{7}$

$8^{2x+1} = 4^{1-x}$

$(2^3)^{2x+1} = (2^2)^{1-x}$

$2^{6x+3} = 2^{2-2x}$

$6x+3 = 2-2x$

$8x = -1$

$x = -\frac{1}{8}$

Properties of Logs/Natural Logs

Why?

Product Property: $\log_b(xy) = \log_b x + \log_b y$

$\ln(xy) = \ln x + \ln y$

$\log_b(xy) = \log_b x + \log_b y$

$b^{\log_b(xy)} = b^{\log_b x + \log_b y}$

$xy = b^{\log_b x} \cdot b^{\log_b y}$

$\checkmark \; = x \cdot y$

$x^2 \cdot x^3 = x^{2+3} = x^5$

Quotient Property: $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

$\ln\left(\frac{x}{y}\right) = \ln x - \ln y$

$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

$b^{\log_b\left(\frac{x}{y}\right)} = b^{\log_b x - \log_b y}$

$\frac{x}{y} = b^{\log_b x} / b^{\log_b y}$

$\checkmark \; = \frac{x}{y}$

$\frac{x^5}{x^3} = x^{5-3} = x^2$

Power Property: $\log_b x^c = c \log_b x$

$\ln x^c = c \ln x$

Let $m = \log_b x$, then

$b^m = x$

$(b^m)^c = x^c$

$b^{mc} = x^c \rightarrow \log_b x^c = mc$

so, $\log_b x^c = (\log_b x) \cdot c$

Change of Base: $\log_b x = \frac{\log_a x}{\log_a b}$

$\log_b x = \frac{\ln x}{\ln b}$

Let $\log_b x = r$, then

$b^r = x$

$\log_a b^r = \log_a x$

$r \log_a b = \log_a x \longrightarrow r = \frac{\log_a x}{\log_a b}$

Examples

Using the properties of logarithms, expand the logarithmic expression.

1. $\ln 3x$

$\ln(3\cdot x)$ *product

$\boxed{\ln 3 + \ln x}$

2. $\log\left(\frac{4x}{y^2}\right)$

$\log(4x) - \log y^2$ *quotient

$\log 4 + \log x - \log y^2$ *product

$\boxed{\log 4 + \log x - 2\log y}$ *power

3. $\log_2 25x^3$

$\log_2 25 + \log_2 x^3$ *product

$\log_2 5^2 + \log_2 x^3$

$\boxed{2\log_2 5 + 3\log_2 x}$ *power

4. $\log \sqrt[3]{\frac{x^2}{y}}$

$\log\left(\frac{x^2}{y}\right)^{1/3}$ *rewrite + simplify

$\log\left(\frac{x^{2/3}}{y^{1/3}}\right)$

$\log x^{2/3} - \log y^{1/3}$ *quotient

$\boxed{\frac{2}{3}\log x - \frac{1}{3}\log y}$ *power

Using the properties of logarithms, condense the logarithms into a single expression.

5. $\log x + 3\log y$

$\log x + \log y^3$ *power

$\boxed{\log(xy^3)}$ *product

6. $\ln 4x - \ln 2y$

$\ln\left(\frac{4x}{2y}\right)$ *quotient

$\boxed{\ln\left(\frac{2x}{y}\right)}$ *simplify

7. $2\log x - \frac{1}{3}\log y + \log a$

$\log x^2 - \log y^{1/3} + \log a$ *power

$\log\left(\frac{x^2}{y^{1/3}}\right) + \log a$ *quotient

$\boxed{\log\left(\frac{ax^2}{y^{1/3}}\right)}$ *product

Write the expression as a natural logarithm.

8. $\log_5 x$

$\boxed{\frac{\ln x}{\ln 5}}$ *change of base

9. $\log_4(2x + y)$

$\boxed{\frac{\ln(2x+y)}{\ln 4}}$ *change of base