DATE: _____

3.5 Equation Solving & Modeling (Target 3B)

Review of Prior Concepts (ACT Warmup)

1. If $\log_5 x = 2$, what is $\sqrt{x}$?

(A) 5
(B) 25
(C) 32
(D) $\sqrt{5}$
(E) $4\sqrt{2}$

$5^{\log_5 x} = 5^2$

$x = 25$

$\sqrt{x} = \sqrt{25}$

$= 5$

2. If $8^{n-1} = \sqrt{2}$, then $n = ?$

(F) $\frac{3}{2}$
(G) $\frac{4}{3}$
(H) $\frac{5}{4}$
(J) $\frac{7}{6}$
(K) $\frac{11}{10}$

$8^{n-1} = \sqrt{2}$

$(2^3)^{n-1} = 2^{1/2}$

$2^{3n-3} = 2^{1/2}$

$3n - 3 = \frac{1}{2}$

$3n = \frac{7}{2}$

$n = \frac{7}{6}$

Find the solution algebraically.

1. $32\left(\frac{1}{4}\right)^{\frac{x}{3}} = 2$

$\left(\frac{1}{4}\right)^{x/3} = \frac{2}{32}$ * divide by 32

$\left(\frac{1}{4}\right)^{x/3} = \frac{1}{16}$ * reduce

$(4^{-1})^{x/3} = 4^{-2}$ * rewrite w/ base 4

$4^{-x/3} = 4^{-2}$

$-\frac{x}{3} = -2$ * bases =, so exponents =

$-x = -6$

$x = 6$

2. $3 \cdot 2^x = 48$

$2^x = 16$ * divide by 3

$2^x = 2^4$ * rewrite w/ base 2

$x = 4$ * bases =, exp. =.

3. $0.35^x = 8$

$\ln 0.35^x = \ln 8$ * Log/ln both side

$x \ln 0.35 = \ln 8$ * power proper

$x = \frac{\ln 8}{\ln 0.35}$ * divide

$x = -1.981$

4. $2 \cdot 10^{2x} = 14$

$10^{2x} = 7$ * divide by 2

$\log 10^{2x} = \log 7$ * log both sides

$2x = \log 7$ * divide by 2

$x = \frac{\log 7}{2}$

$x = 0.423$

5. $3 + 2e^{-x} = 6$

$2e^{-x} = 3$ * subtract 3

$e^{-x} = 3/2$ * divide by 2

$\ln e^{-x} = \ln\left(\frac{3}{2}\right)$ * ln both sides

$-x = \ln\left(\frac{3}{2}\right)$ * divide by -1

$x = -\ln\left(\frac{3}{2}\right)$

$x = -.405$

6. $3\ln(x-4) - 2 = 6$

$3\ln(x-4) = 8$ * add 2

$\ln(x-4) = \frac{8}{3}$ * divide by 3

$e^{\ln(x-4)} = e^{8/3}$ * antilog "e" both sides

$x - 4 = e^{8/3}$ * add 4

$x = e^{8/3} + 4$

$x = 18.392$

7. $\log x - \log(x+4) = 1$

$\log\left(\frac{x}{x+4}\right) = 1$ * quotient property

$10^{\log\left(\frac{x}{x+4}\right)} = 10^{1}$ * antiLog "10" both sides

$\frac{x}{x+4} = 10$ * multiply by $x+4$

$x = 10x + 40$ * solve for x

$-9x = 40$

$x = -\frac{40}{9}$ ← extraneous solution

check

$\log(-\frac{40}{9}) - \log(-\frac{40}{9}+4) \neq 1$

can't take log of - #

∴ no solution

8. $\ln(3x-2) + \ln(x-1) = 2\ln x$

$\ln((3x-2)(x-1)) = \ln x^2$ * product * power

$e^{\ln((3x-2)(x-1))} = e^{\ln x^2}$ * antiLog "e" both sides

$(3x-2)(x-1) = x^2$

$3x^2 - 3x - 2x + 2 = x^2$

$3x^2 - 5x + 2 = x^2$

$2x^2 - 5x + 2 = 0$

$2x^2 - 4x - 1x + 2 = 0$

$2x(x-2) - 1(x-2) = 0$

$(2x-1)(x-2) = 0$

$x = \frac{1}{2}$, $x = 2$

↓ extraneous

$\ln(3\cdot\frac{1}{2} - 2) + \ln(\frac{1}{2} - 1) = 2\ln\frac{1}{2}$

$\ln(-\frac{1}{2}) + \ln(-\frac{1}{2})$

can't take ln of - # ... not in domain of ln

9. $\frac{2^x + 5\cdot 2^{-x}}{2} = 3$

$2^x + 5\cdot 2^{-x} = 6$ * multiply by 2

$2^x + 5(\frac{1}{2^x}) = 6$ * rewrite neg exp.

$2^x \cdot 2^x + 5(\frac{1}{2^x})\cdot 2^x = 6\cdot 2^x$ * multiply by LCD

$(2^x)^2 + 5 = 6\cdot 2^x$

$(2^x)^2 - 6\cdot 2^x + 5 = 0$

let $u = 2^x$

$u^2 - 6u + 5 = 0$

$(u-5)(u-1) = 0$

$u = 5$, $u = 1$

$2^x = 5$ $\quad 2^x = 1$ * replace back 2^x for u

$\ln 2^x = \ln 5$ $\quad \ln 2^x = \ln 1$ * solve for x

$x\ln 2 = \ln 5$ $\quad x\ln 2 = 0$

$x = \frac{\ln 5}{\ln 2}$ $\quad x = 0$

$x = 2.322$