

Average Rate of Change

Target 5E. Calculate and interpret the average rate of change of a function as the function relates to a real world situation.

Do you remember how to find the slope of a line passing between two points?

Slope $m = \frac{y_2 - y_1}{x_2 - x_1}$, $\frac{\text{RISE}}{\text{RUN}}$, $\frac{\Delta Y}{\Delta X}$, Given: $(x_1, f(x_1)), (x_2, f(x_2))$
 Slope can be expressed as: $\frac{f(x_2) - f(x_1)}{x_2 - x_1}$

Determine the slope of the line passing between the two given points.

1. $(-1, 4)$ and $(1, -2)$
 $\begin{matrix} x_1 & y_1 & & x_2 & y_2 \end{matrix}$

→ I'll use $m = \frac{y_2 - y_1}{x_2 - x_1}$ here.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 4}{1 - (-1)} = \frac{-6}{2} = \boxed{-3}$$

2. $(1, 3)$ and $(-2, -3)$
 $\begin{matrix} x_1 & y_1 & & x_2 & y_2 \end{matrix}$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 3}{-2 - 1} = \frac{-6}{-3} = \boxed{3}$$

3. $(-1, 4)$ and $(-4, 5)$
 $\begin{matrix} x_2 & y_2 & & x_1 & y_1 \end{matrix}$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 5}{-1 - (-4)} = \frac{-1}{3} = \boxed{-\frac{1}{3}}$$

Average Rate of Change

Use a calculator to graph each function and determine the average rate of change between the given x-values. Recall $y = f(x)$!

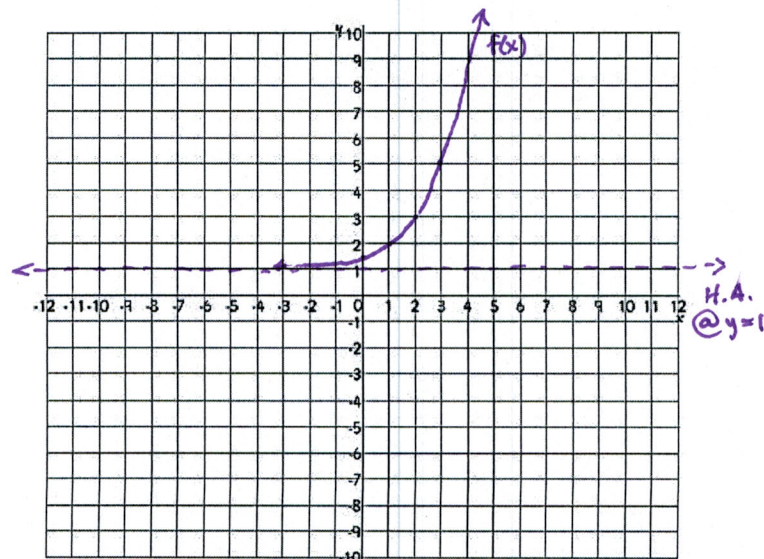
4. $f(x) = \frac{1}{2}(2)^x + 1$ from $x = 2$ to $x = 4$

x	f(x)
-3	1.063
0	1.5
1	2
2	3
3	5
4	9

Avg rate of change

$$\frac{f(4) - f(2)}{4 - 2} = \frac{9 - 3}{4 - 2} = \frac{6}{2} = \boxed{3}$$

$(2, f(2))$
 $(4, f(4))$

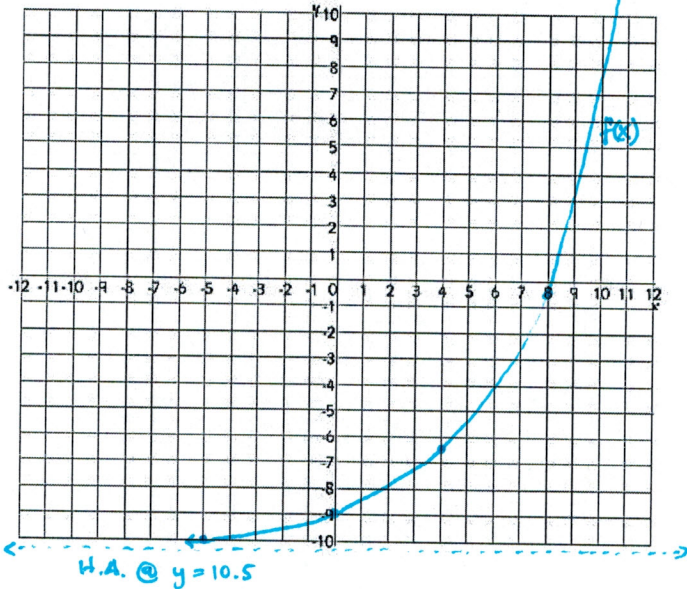


5. $f(x) = \frac{3}{2}(e)^{\frac{1}{4}x} - 10.5$
 from $x = 0$ to $x = 11$

x	f(x)
-5	-10.070
0	-9
4	-6.423
8	0.584
11	12.964

Avg. rate of change
 $\frac{f(11) - f(0)}{11 - 0} = \frac{12.964 - (-9)}{11 - 0}$
 $= \frac{21.964}{11}$
 $= \boxed{1.997}$

$(0, f(0)), (11, f(11))$



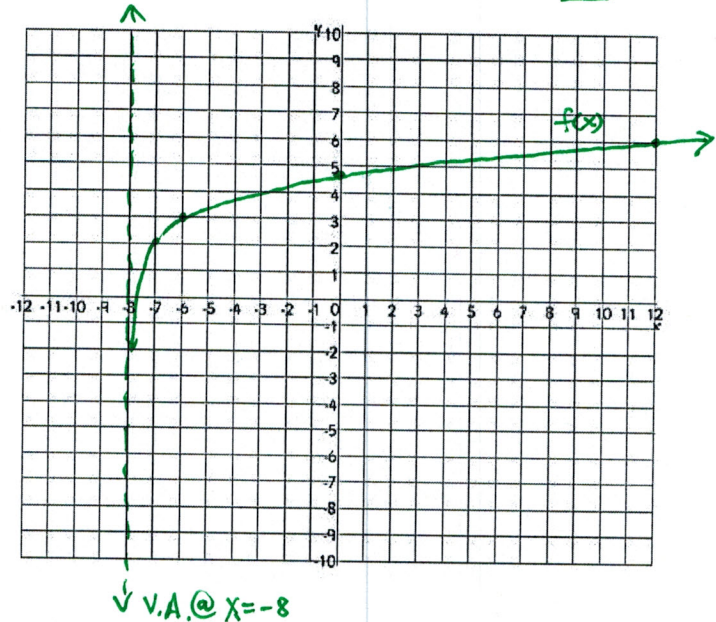
V.A. @ $5x + 40 = 0$
 $5x = -40$
 $x = -8$

6. $f(x) = 3 \log_{10}(5x + 40)$
 from $x = -6$ to $x = 12$

x	f(x)
-7	2.097
-6	3
0	4.806
12	6

Avg. rate of change
 $\frac{f(12) - f(-6)}{12 - (-6)} = \frac{6 - 3}{12 + 6}$
 $= \frac{3}{18}$
 $= \boxed{\frac{1}{6}}$

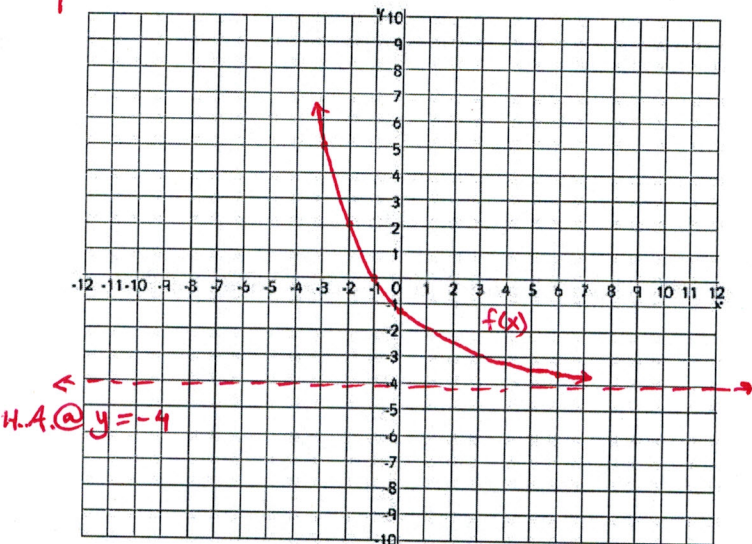
$(-6, f(-6)), (12, f(12))$



7. $f(x) = 6\left(\frac{2}{3}\right)^{x+2} - 4$ $(-3, f(-3))$
 from $x = -3$ to $x = -1$ $(-1, f(-1))$

x	f(x)
-3	5
-2	2
-1	0
0	-1.3
6	-3.766

Avg. rate of change
 $\frac{f(-1) - f(-3)}{-1 - (-3)} = \frac{0 - 5}{-1 + 3}$
 $= \frac{-5}{2}$
 $= \boxed{-\frac{5}{2}}$



8. $f(x) = \log_2(x) - 4$
 from $x = 1$ to $x = 8$

x	f(x)
1	-4
2	-3
4	-2
8	-1
16	0

Avg. rate of change
 $\frac{f(8) - f(1)}{8 - 1} = \frac{-1 - (-4)}{8 - 1}$
 $= \frac{-1 + 4}{8 - 1}$
 $= \frac{3}{7}$
 $= \boxed{\frac{3}{7}}$

$(1, f(1)), (8, f(8))$

