P5-4 #33, 34, 40, 45, 46

More Practice and Problem-Solving Exercises *continued*

23. about 3 dB

24. The coefficient $\frac{1}{2}$ is missing in $\log_4 s$;

$\log_4 \sqrt{\frac{t}{s}} = \frac{1}{2}\log_4 \frac{t}{s}$

$= \frac{1}{2}(\log_4 t - \log_4 s)$

$= \frac{1}{2}\log_4 t - \frac{1}{2}\log_4 s$

25. No; the expression $(2x + 1)$ is a sum, so it is not covered by the Product, Quotient, or Power Properties.

26. The log of a product is equal to the sum of the logs. $\log(MN) = \log M + \log N$.

27. true; $\log_2 4 = 2$ and $\log_2 8 = 3$; $2 + 3 = 5$

28. false; $\frac{1}{2}\log_3 3 = \log_3 3^{\frac{1}{2}}$, not $\log_3 \frac{3}{2}$

29. False; this is not an example of the Quotient Properties $\log(x - 2) \neq \log x - \log 2$.

30. false; $\log_b \frac{x}{y} = \log_b x - \log_b y$, while $\frac{\log_b x}{\log_b y} = \log_y x$

31. false; $(\log x)^2 = (\log x) \cdot (\log x)$, while $\log x^2 = 2\log x = (\log x) + (\log x)$

32. false; $\log_4 7 - \log_4 3 = \log_4 \frac{7}{3}$ not $\log_4(7 - 3)$

33. $\log_3 \sqrt[4]{2x}$

34. $\log_x \frac{2\sqrt{y}}{z^3}$

35. $\log_4 \frac{m^x \sqrt[y]{n}}{p}$

36. $\log_b \frac{\sqrt[3]{x^2}\sqrt[4]{y^3}}{z^5}$

37. $\frac{1}{2}\log 2 + \frac{1}{2}\log x - \frac{1}{2}\log y$

38. $\log s + \frac{1}{2}\log 7 - 2\log t$

39. $3\log 2 + \frac{3}{2}\log x - 3\log 5$

40. $3\log m - 4\log n + 2\log p$

41. $3\log 2 + \frac{1}{2}\log r - \log s$

42. $\frac{1}{2}\log_b x + \frac{2}{3}\log_b y - \frac{2}{5}\log_b z$

43. $\frac{5}{2}\log_4 x + \frac{7}{2}\log_4 y - \log_4 z - 4\log_4 w$

44. $\frac{1}{2}\log(x + 2) + \frac{1}{2}\log(x - 2) - 2\log(x + 3)$

45. $\frac{\log 2}{\log 7}$ **46.** $\frac{\log 8}{\log 3}$

47. $\frac{\log 140}{\log 5}$ **48.** $\frac{\log 3.3}{\log 9}$

49. $\frac{\log 3x}{\log 4}$

50. A 1.0 magnitude star is about 2.5 times brighter than a 2.0 magnitude star.

51. Capella is about 1.02 times brighter than Rigel.

52. $\frac{1}{2}\log x + \frac{1}{4}\log 2 - \log y$

53. $3\log_3 [xy^{\frac{1}{3}} + z^2]$

STEM **23. Construction** The foreman of a construction team puts up a sound barrier that reduces the intensity of the noise by 50%. By how many decibels is the noise reduced? Use the formula $L = 10\log\frac{I}{I_0}$ to measure loudness. (*Hint:* Find the difference between the expression for loudness for intensity I and the expression for loudness for intensity $0.5I$.)

24. Error Analysis Explain why the expansion at the right of $\log_4\sqrt{\frac{t}{s}}$ is incorrect. Then do the expansion correctly.

$\log_4\sqrt{\frac{t}{s}} = \frac{1}{2}\log_4\frac{t}{s}$
$= \frac{1}{2}\log_4 t - \log_4 s$

25. Reasoning Can you expand $\log_3(2x + 1)$? Explain.

26. Writing Explain why $\log(5 \cdot 2) \neq \log 5 \cdot \log 2$.

Determine if each statment is *true* or *false*. Justify your answer.

27. $\log_2 4 + \log_2 8 = 5$

28. $\log_3 \frac{3}{2} = \frac{1}{2}\log_3 3$

29. $\log(x - 2) = \frac{\log x}{\log 2}$

30. $\frac{\log_b x}{\log_b y} = \log_b \frac{x}{y}$

31. $(\log x)^2 = \log x^2$

32. $\log_4 7 - \log_4 3 = \log_4 4$

Write each logarithmic expression as a single logarithm.

33. $\frac{1}{4}\log_3 2 + \frac{1}{4}\log_3 x$

34. $\frac{1}{2}(\log_x 4 + \log_x y) - 3\log_x z$

35. $x\log_4 m + \frac{1}{y}\log_4 n - \log_4 p$

36. $\left(\frac{2\log_b x}{3} + \frac{3\log_b y}{4}\right) - 5\log_b z$

Expand each logarithm.

37. $\log\sqrt{\frac{2x}{y}}$

38. $\log\frac{s\sqrt{7}}{t^2}$

39. $\log\left(\frac{2\sqrt{x}}{5}\right)^3$

40. $\log\frac{m^3}{n^4p^{-2}}$

41. $\log 4\sqrt{\frac{4r}{s^2}}$

42. $\log_b \frac{\sqrt{x}\sqrt[3]{y^2}}{\sqrt[5]{z^2}}$

43. $\log_4 \frac{\sqrt{x^5y^7}}{zw^4}$

44. $\log\frac{\sqrt{x^2 - 4}}{(x + 3)^2}$

Write each logarithm as the quotient of two common logarithms. Do not simplify the quotient.

45. $\log_7 2$ **46.** $\log_3 8$ **47.** $\log_5 140$ **48.** $\log_9 3.3$ **49.** $\log_4 3x$

STEM **Astronomy The apparent brightness of stars is measured on a logarithmic scale called magnitude, in which lower numbers mean brighter stars. The relationship between the ratio of apparent brightness of two objects and the difference in their magnitudes is given by the formula $m_2 - m_1 = -2.5\log\frac{b_2}{b_1}$, where m is the magnitude and b is the apparent brightness.**

50. How many times brighter is a magnitude 1.0 star than a magnitude 2.0 star?

51. The star Rigel has a magnitude of 0.12. How many times brighter is Capella than Rigel?

Challenge

Expand each logarithm.

52. $\log\sqrt{\frac{x\sqrt{2}}{y^2}}$

53. $\log_3[(xy^{\frac{1}{3}}) + z^2]^3$

54. $\log_7\frac{\sqrt{r + 9}}{s^2t^{\frac{1}{3}}}$

Simplify each expression.

55. $\log_3(x + 1) - \log_3(3x^2 - 3x - 6) + \log_3(x - 2)$

56. $\log(a^2 - 10a + 25) + \frac{1}{2}\log\frac{1}{(a - 5)^3} - \log(\sqrt{a - 5})$

54. $\frac{1}{2}\log_7(r + 9) - 2\log_7 s - \frac{1}{3}\log_7 t$

55. -1

56. 0

P5-4 # 33, 34, 40, 45, 46

#33 $\frac{1}{4}\log_3 2 + \frac{1}{4}\log_3 x$

$\log_3 2^{\frac{1}{4}} + \log_3 x^{\frac{1}{4}}$

$\log 2^{\frac{1}{4}} \cdot x^{\frac{1}{4}}$

$\log (2x)^{\frac{1}{4}}$

$\boxed{\log \sqrt[4]{2x}}$

#34 $\frac{1}{2}(\log_x 4 + \log_x y) - 3\log_x z$

$\frac{1}{2}\log_x 4 + \frac{1}{2}\log_x y - 3\log_x z$

$\log_x 4^{\frac{1}{2}} + \log_x y^{\frac{1}{2}} - \log_x z^3$

$\log_x 4^{1/2} = \log_x \sqrt{4}$

$\log_x 2 + \log_x \sqrt{y} - \log_x z^3$

$\log_x 2\sqrt{y} - \log_x z^3$

$\boxed{\log_x \frac{2\sqrt{y}}{z^3}}$

#40 $\log \frac{m^3}{n^4 p^{-2}} = \log \frac{m^3 p^2}{n^4}$

No negative exponents: $p^{-2} = \frac{1}{p^2}$ $\frac{1}{p^{-2}} = p^2$

$\log m^3 p^2 - \log n^4$

$\log m^3 + \log p^2 - \log n^4$

$\boxed{3\log m + 2\log p - 4\cdot\log n}$

or

$\boxed{3\cdot\log m - 4\log n + 2\log p}$

#45 $\log_7 2 = \boxed{\frac{\log 2}{\log 7}}$

#46 $\log_3 8 = \boxed{\frac{\log 8}{\log 3}}$