P5-7: Geometric Sequences & Series
Honors Advanced Algebra

Name: _____
Period: _____ Date: _____

Is the sequence geometric? If it is, what are a_1 and r?

1. $2, 4, 8, 16, \ldots$
$\frac{4}{2} = \frac{8}{4} = \frac{16}{8} = 2$ Yes. $a_1 = 2$, $r = 2$

2. $1, 5, 9, 13, 17, \ldots$ No
$5 = \frac{5}{1} \neq \frac{9}{5}$

3. $2^3, 2^7, 2^{11}, 2^{15}, \ldots$
$\frac{2^7}{2^3} = \frac{2^{11}}{2^7} = \frac{2^{15}}{2^{11}} = 2^4$ Yes. $a_1 = 2^3$, $r = 2^4$

4. $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$ No
$\frac{1}{2} = \frac{\frac{1}{2}}{1} \neq \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{1}{3} \cdot \frac{2}{1} = \frac{2}{3}$

What are the indicated terms of the geometric sequence?

5. The second term of the geometric sequence $3, \underline{-6}\text{ (or } 6\text{)}, 12, \ldots$
$a_1 = 3$, $a_3 = 12$, $a_n = a_1 \cdot r^{n-1}$, $a_2 = ?$
$a_3 = a_1 \cdot r^{3-1}$
$a_3 = a_1 \cdot r^2$
$12 = 3 \cdot r^2$
$4 = r^2$
$r = \pm 2$. Now, use $a_n = a_{n-1} \cdot r$
$a_2 = a_{2-1} \cdot r = a_1 \cdot r = 3(2) = \boxed{6}$ or
$a_2 = 3(-2) = \boxed{-6}$

6. The ~~eighth~~ (Yes, I want eighth ... sorry) term of the geometric sequence $10, 5, 2.5, \ldots$
$a_1 = 10$
$\frac{5}{10}$, $\frac{2.5}{5}$
$r = \frac{5}{10} = \frac{2.5}{5} = \frac{1}{2}$
$a_n = a_1 \cdot r^{n-1} \Rightarrow a_8 = a_1 \cdot r^{8-1} = a_1 \cdot r^7 = 10\left(\frac{1}{2}\right)^7 = \frac{10}{2^7} = \boxed{\frac{5}{64}}$
$\therefore$ Eighth term is $\boxed{a_8 = \frac{5}{64}}$

7. When radioactive substances decay, the amount remaining will form a geometric sequence when measured over constant intervals of time. The table below shows the amount of Np-240, a radioactive isotope of Neptunium, initially and after 2 hours. What are the amounts left after 1 hour, 3 hours and 4 hours?

Hours Elapsed	0	1	2	3	4
Grams of Np-240	1244	655.588 ≈ 656	346	182.342 ≈ 182	95.914 ≈ 96
	a_1	a_2	a_3	a_4	a_5

$a_1 = 1244$, $a_3 = 346$
$a_n = a_1 \cdot r^{n-1}$
$a_3 = a_1 \cdot r^{3-1}$
$a_3 = a_1 \cdot r^2 \Rightarrow \frac{346}{1244} = \frac{1244 \cdot r^2}{1244}$
$\Rightarrow \frac{346}{1244} = r^2$
$\therefore r \approx 0.527$

$a_2 = a_{2-1} \cdot r = a_1 \cdot r = 1244(0.527) = 655.588 \approx \boxed{656}$
$a_4 = a_{4-1} \cdot r = a_3 \cdot r = 346(0.527) = 182.342 \approx \boxed{182}$
$a_5 = a_{5-1} \cdot r = a_4 \cdot r = 182(0.527) \approx 95.914 \approx \boxed{96}$

Evaluate the sum of the finite geometric series.

8. $-5 - 10 - 20 - 40 - \cdots - 2560$
$\frac{-10}{-5} = \frac{-20}{-10} = \frac{-40}{-20} = 2$
$a_1 = -5$, $r = 2$
nth term: $a_n = -2560$
$a_n = a_1 r^{n-1}$
$-2560 = -5 \cdot 2^{n-1}$
$512 = 2^{n-1}$
$2^9 = 2^{n-1}$
$9 = n - 1$
$10 = n$
$S_n = \frac{a_1(1 - r^n)}{1 - r}$
$= \frac{-5(1 - 2^{10})}{1 - 2}$
$= \boxed{-5115}$

9. $\frac{1}{5} + \frac{1}{10} + \frac{1}{20} + \frac{1}{40} + \frac{1}{80}$
$a_1 = \frac{1}{5}$
$\frac{\frac{1}{10}}{\frac{1}{5}} = \frac{1}{10} \cdot \frac{5}{1} = \frac{1}{2}$, $\frac{\frac{1}{20}}{\frac{1}{10}} = \frac{1}{20} \cdot \frac{10}{1} = \frac{1}{2} \ldots$
So $r = \frac{1}{2}$
There are 5 terms:
$S_5 = \frac{\frac{1}{5}\left(1 - \left(\frac{1}{2}\right)^5\right)}{1 - \frac{1}{2}} = \boxed{0.3875}$

10. $9 - 6 + 4 - \frac{8}{3} + \frac{16}{9}$
$a_1 = 9$
$\frac{-6}{9} = \frac{4}{-6} = \frac{-\frac{8}{3}}{4} = \frac{\frac{16}{9}}{-\frac{8}{3}} = -\frac{2}{3}$
$r = -\frac{2}{3}$
$S_5 = \frac{9\left(1 - \left(-\frac{2}{3}\right)^5\right)}{1 - \left(-\frac{2}{3}\right)} = \boxed{\frac{55}{9}}$