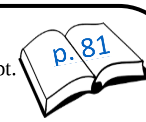
1.2 Functions and Their Properties
Domain, Range, & Continuity of Functions
Target 1A: Analyze functions using specific properties

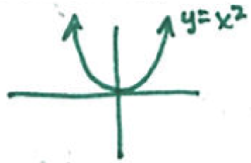
Review of Prior Concepts

Is the formula a function? (Graph them to complete the vertical line test).



Vertical Line Test: A graph in the coordinate plane defines *y* as a function of *x* ⇔ no vertical line intersects the graph in more than one pt.

1. $y = x^2$



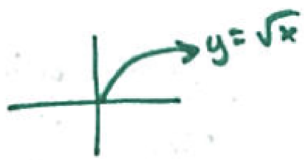
∴ → therefore

passes vertical line test
∴, y=x² is a function

2. $y^2 = x$

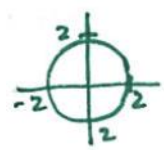
fails vertical line test
∴, y²=x is NOT a function

3. $y = \sqrt{x}$



passes vertical line test
∴, y=√x is a function

4. $x^2 + y^2 = 4$



x²+y²=r² circle

fails vertical line test
∴, x²+y²=4 is NOT a function

A *function* from a set D to a set R (f: D → R) is a rule that assigns to every element in D a unique element in R; i.e., The set D of all input values is the *Domain* of the function, and the set R of all output values is the *Range* of the function.

More Practice

Is it a Function?
http://www.mathwarehouse.com/algebra/relation/vertical-line-test.php
https://www.youtube.com/watch?v=zT69oxcMhPw
https://www.khanacademy.org/math/cc-eighth-grade-math/cc-8th-linear-equations-functions/cc-8th-function-intro/e/recog-func-2

SAT Connection

Passport to Advanced Math

13. Use function notation, and interpret statements using function notation.

Example:

$$g(x) = ax^2 + 24$$

For the function g defined above, a is a constant and $g(4) = 8$. What is the value of $g(-4)$?

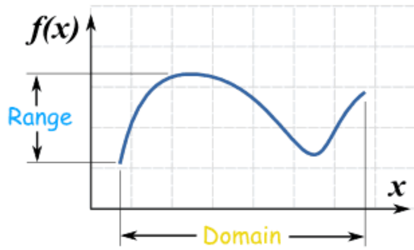
A) 8
B) 0
C) −1
D) −8

$g(4) = a(4)^2 + 24$
$8 = 16a + 24$
$-16 = 16a$
$-1 = a$

$g(x) = -x^2 + 24$
$g(-4) = -(-4)^2 + 24$
$= -16 + 24$
$= 8$

Solution

Domain & Range



Domain → x-values
input values

Range → y-values
output/results

Find the domain algebraically & the range graphically of each function.

p.81 – p.84

Example 1:

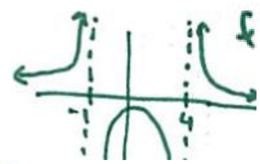
$f(x) = \frac{2}{x^2-3x-4}$

Domain

can't divide by zero, so
set denominator ≠ to zero
$x^2-3x-4 \neq 0$
$(x-4)(x+1) \neq 0$
$x \neq 4,\ x \neq -1$

$\boxed{\text{Domain: } (-\infty,-1) \cup (-1,4) \cup (4,\infty)}$

Range



$\boxed{\text{Range: } (-\infty, -0.32] \cup (0, \infty)}$

Example 2:

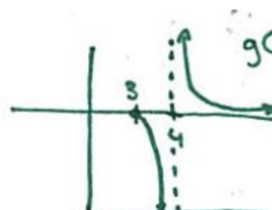
$g(x) = \frac{\sqrt{x-3}}{x^2-3x-4}$

Domain

see above, $x \neq -1,\ x \neq 4$
also, can't $\sqrt{\ }$ negative #s
so, $x-3 \geq 0$
$x \geq 3$

$\boxed{\text{Domain: } [3,4) \cup (4,\infty)}$

Range



$\boxed{\text{Range: } (-\infty,\infty)}$

Example 3:

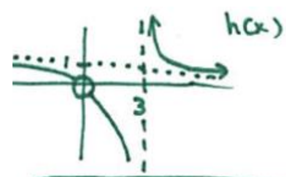
$h(x) = \frac{x^2}{x^2-3x}$

Domain

$x^2-3x \neq 0$
$x(x-3) \neq 0$
$x \neq 0,\ x-3 \neq 0$
$x \neq 3$

$\boxed{\text{Domain: } (-\infty,0) \cup (0,3) \cup (3,\infty)}$

Range



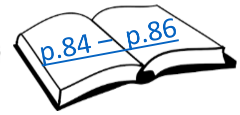
$\boxed{\text{Range: } (-\infty,0) \cup (0,1) \cup (1,\infty)}$

More Practice

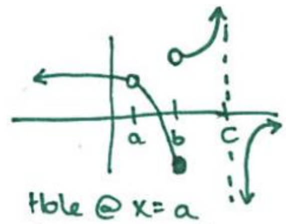
Domain & Range

http://www.coolmath.com/algebra/15-functions/06-finding-the-domain-01
https://www.khanacademy.org/math/algebra/algebra-functions/domain-and-range/v/domain-of-a-function-intro
http://www.intmath.com/functions-and-graphs/2a-domain-and-range.php

Continuity & Discontinuity



- Functions are continuous if there are no jumps, holes or asymptotes
  (no breaks in the graphs)

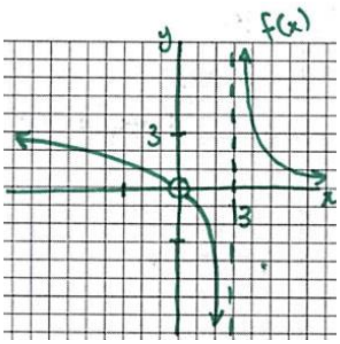


- Removable discontinuity
  (can make the discontinuity go away)
  HOLE in the graph @ $x = a$
- Non-removable discontinuity (can't make discont. go away)
  - JUMP
  - INFINITE (vertical asymptote)

Hole @ $x=a$
Jump @ $x=b$
Infinite (V.A.) @ $x=c$

Graph the function. Identify any points of discontinuity and describe the type of discontinuity.

Example 4: $f(x) = \frac{x^2}{x^2-3x}$



infinite discontinuity (non-removable)
@ $x = 3$
hole @ $x=0$ (removable)

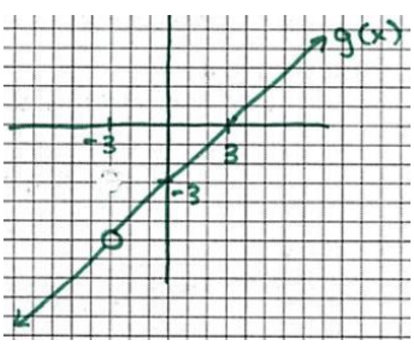
check algebraically

$f(x) = \frac{x^2}{x^2-3x}$

$= \frac{\cancel{x} \cdot x}{\cancel{x}(x-3)} \rightarrow$ non-removable: $x-3=0$, $x=3$ (V.A.)

removable @ $x=0$ (hole)

Example 5: $g(x) = \frac{x^2-9}{x+3}$



hole @ $x = -3$ (removable)

check algebraically

$g(x) = \frac{x^2-9}{x+3}$

$= \frac{(x-3)\cancel{(x+3)}}{\cancel{x+3}} \rightarrow$ removable: $x+3=0$, $x=-3$ (hole)

$= x-3$

More Practice

Continuity

http://www.ck12.org/Analysis/Discrete-and-Continuous-Functions/lesson/Continuity-and-Discontinuity-PCALC/

https://www.youtube.com/watch?v=2n5VzMFJQVY

Homework Assignment

p.94 #1,3,13,14,15,18,19

SAT Connection

Solution

Choice A is correct. Since g is an even function, $g(-4) = g(4) = 8$.

Alternatively: First find the value of a, and then find $g(-4)$. Since $g(4) = 8$, substituting 4 for x and 8 for $g(x)$ gives $8 = a(4)^2 + 24 = 16a + 24$. Solving this last equation gives $a = -1$. Thus $g(x) = -x^2 + 24$, from which it follows that $g(-4) = -(-4)^2 + 24$; $g(-4) = -16 + 24$; and $g(-4) = 8$.

Choices B, C, and D are incorrect because g is a function and there can only be one value of $g(-4)$.