

## 1.2 Functions and Their Properties

### Symmetry, End Behavior of Functions

Target 1A: Analyze functions using specific properties

*Review of Prior Concepts*

Which of the letters of the alphabet have vertical symmetry?

(Hint: A is one of them)

A B C D E F  
G H I J K L  
M N O P Q  
R S T U V  
W X Y Z

Which have  $180^\circ$  rotational symmetry?

### More Practice

**Symmetry**

<http://gwydir.demon.co.uk/jo/symmetry/refsym.htm>

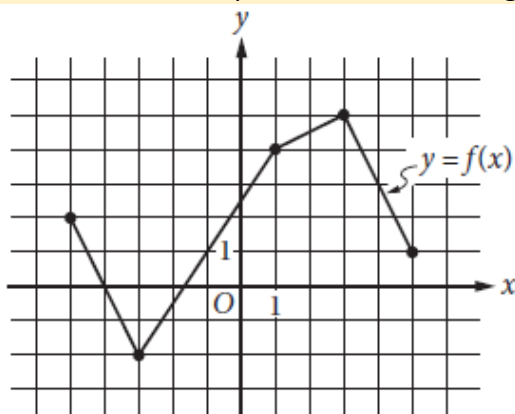
<https://www.khanacademy.org/math/geometry/transformations/transformations-symmetry/v/example-rotating-polygons>

### SAT Connection

#### Passport to Advanced Math

**13.** Use function notation, and interpret statements using function notation.

Example:



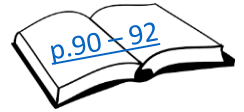
The complete graph of the function  $f$  is shown in the  $xy$ -plane above. For what value of  $x$  is the value of  $f(x)$  at its minimum?

- A)  $-5$
- B)  $-3$
- C)  $-2$
- D)  $3$

[Solution](#)

## Symmetry

- Even Functions – (graphically)
  - (numerically)
  - (algebraically)
- Odd Functions – (graphically)
  - (numerically)
  - (algebraically)



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Determine graphically whether the function is even, odd, or neither. Check algebraically.

*Example 1:*  $f(x) = 2x^4 + x^2 + 1$

*Example 2:*  $g(x) = 2x^3 + x + 1$

*Example 3:*  $h(x) = 2x^3 + x$

### End Behavior

End Behavior – what happens at the ends of the function.

*NOTATION:*

Describe the end behavior of the function from the graph of the function.

*Example 4:*  $f(x) = 2x^4 + x^2 + 1$

*Example 5:*  $g(x) = 2x^3 + x + 1$

*Example 6:*  $h(x) = \frac{x}{x^2+2}$

*Example 7:*  $r(x) = \frac{x^3}{x^2+2}$

*Example 8:*  $p(x) = \frac{x^2}{x^2+2}$

Horizontal Asymptotes – occur when end behavior approaches a #,  $c$ . H.A. is @  $y = c$ .

*NOTATION:*  $\lim_{x \rightarrow \infty} f(x) = c$  or  $\lim_{x \rightarrow -\infty} f(x) = c$

#### More Practice

##### Symmetry

<https://www.chilimath.com/algebra/intermediate/oef/even-and-odd-functions.html>

<https://www.youtube.com/watch?v=1LsJaR72UFM>

##### End Behavior

<http://www.coolmath.com/precalculus-review-calculus-intro/precalculus-algebra/14-tail-behavior-limits-at-infinity-02>

[https://www.youtube.com/watch?v=Krjd\\_vU4Uvg](https://www.youtube.com/watch?v=Krjd_vU4Uvg)

#### Homework Assignment

p.95 #35,38,39,45,49,50,51,53

**SAT Connection****Solution**

**Choice B is correct.** The minimum value of the function corresponds to the  $y$ -coordinate of the point on the graph that is the lowest along the vertical or  $y$ -axis. Since the grid lines are spaced 1 unit apart on each axis, the lowest point along the  $y$ -axis has coordinates  $(-3, -2)$ . Therefore, the value of  $x$  at the minimum of  $f(x)$  is  $-3$ .

Choice A is incorrect;  $-5$  is the smallest value for an  $x$ -coordinate of a point on the graph of  $f$ , not the lowest point on the graph of  $f$ . Choice C is incorrect; it is the minimum value of  $f$ , not the value of  $x$  that corresponds to the minimum of  $f$ . Choice D is incorrect; it is the value of  $x$  at the maximum value of  $f$ , not at the minimum value of  $f$ .