1.2 Functions and Their Properties
Symmetry, End Behavior of Functions
Target 1A: Analyze functions using specific properties

Review of Prior Concepts

Which of the letters of the alphabet have vertical symmetry?
(Hint: A is one of them)

A, H, I, M, O, T, U, V, W, Y

Which have 180° rotational symmetry?
→ same image upside down

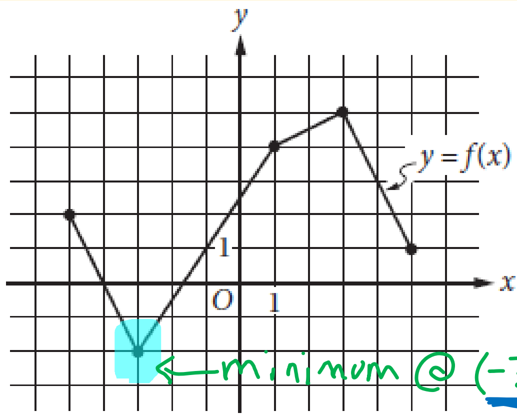
H, I, N, O, S, X, Z

A B C D E F
G H I J K L
M N O P Q
R S T U V
W X Y Z

More Practice

Symmetry
http://gwydir.demon.co.uk/jo/symmetry/refsym.htm
https://www.khanacademy.org/math/geometry/transformations/transformations-symmetry/v/example-rotating-polygons

SAT Connection
Passport to Advanced Math
13. Use function notation, and interpret statements using function notation.
Example:



← minimum @ (−3, −2)

The complete graph of the function f is shown in the xy-plane above. For what value of x is the value of $f(x)$ at its minimum?

$x = -3$

A) −5
B) −3
C) −2
D) 3

Solution

Symmetry

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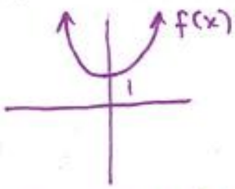
- Even Functions – (graphically) symmetrical about y-axis
  - – (numerically) y-values for positive x is same as y-values for negative x
  - – (algebraically) $f(-x) = f(x)$
- Odd Functions – (graphically) symmetrical about the origin
  - – (numerically) y-values for positive x is opposite as y-values for negative x
  - – (algebraically) $f(x) = -f(x)$

Determine graphically whether the function is even, odd, or neither. Check algebraically.

ex: $f(x) = 2x^4 + x^2 + 1$



f(x) is even b/c f(x) is symmetrical about y-axis

algebraically

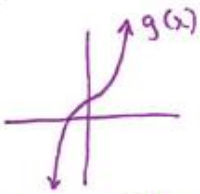
$f(-x) = 2(-x)^4 + (-x)^2 + 1$

$= 2x^4 + x^2 + 1$

same as f(x), so f(x) is even.

*replace −x for every x

ex: $g(x) = 2x^3 + x + 1$



g(x) is neither odd nor even b/c g(x) is not symmetrical about y-axis nor origin.

algebraically

$f(-x) = 2(-x)^3 + (-x) + 1$

$= -2x^3 - x + 1$

not same as f(x)

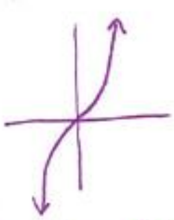
*replace −x for every x

$f(-x) = -(\underbrace{2x^3 - x - 1}_{\text{not same as } f(x)})$

*try factoring out "−"

∴, f(x) is neither odd nor even

ex: $h(x) = 2x^3 + x$



h(x) is odd b/c h(x) is symmetrical about the origin.

algebraically

$f(-x) = 2(-x)^3 + (-x)$

$= -2x^3 - x$

not same as f(x)

*replace −x for every x

$f(-x) = -(\underbrace{2x^3 + x}_{\text{same as } f(x)})$

*try factoring out "−"

so, $f(-x) = -f(x)$

∴, f(x) is odd.

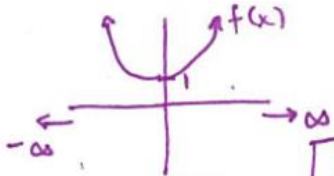
End Behavior

End Behavior – what happens at the ends of the function.

NOTATION: $\lim\limits_{x\to\infty} f(x)$ or $\lim\limits_{x\to-\infty} f(x)$

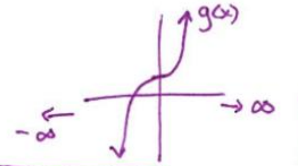
Describe the end behavior of the function from the graph of the function.

Example 4: $f(x) = 2x^4 + x^2 + 1$



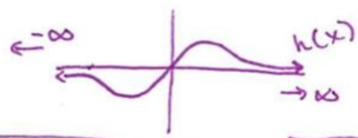
$\boxed{\lim\limits_{x\to-\infty} f(x) = \infty}$ $\boxed{\lim\limits_{x\to\infty} f(x) = \infty}$

Example 5: $g(x) = 2x^3 + x + 1$



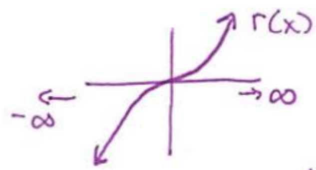
$\boxed{\lim\limits_{x\to-\infty} g(x) = -\infty}$ $\boxed{\lim\limits_{x\to\infty} g(x) = \infty}$

Example 6: $h(x) = \frac{x}{x^2+2}$



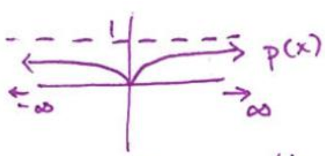
$\boxed{\lim\limits_{x\to-\infty} h(x) = 0}$ $\boxed{\lim\limits_{x\to\infty} h(x) = 0}$

Example 7: $r(x) = \frac{x^3}{x^2+2}$



$\lim\limits_{x\to-\infty} r(x) = -\infty$ $\lim\limits_{x\to\infty} r(x) = \infty$

Example 8: $p(x) = \frac{x^2}{x^2+2}$



$\lim\limits_{x\to-\infty} p(x) = 1$ $\lim\limits_{x\to\infty} p(x) = 1$

Horizontal Asymptotes – occur when end behavior approaches a #, c. H.A. is @ $y = c$.

NOTATION: $\lim\limits_{x\to\infty} f(x) = c$ or $\lim\limits_{x\to-\infty} f(x) = c$

since $\lim\limits_{x\to\infty} p(x) = 1$, there is a horizontal asymptote @ $y = 1$

More Practice

Symmetry

https://www.chilimath.com/algebra/intermediate/oef/even-and-odd-functions.html

https://www.youtube.com/watch?v=1LsJaR72UFM

End Behavior

http://www.coolmath.com/precalculus-review-calculus-intro/precalculus-algebra/14-tail-behavior-limits-at-infinity-02

https://www.youtube.com/watch?v=Krjd_vU4Uvg

Homework Assignment

p.95 #35,38,39,45,49,50,51,53

SAT Connection

Solution

Choice B is correct. The minimum value of the function corresponds to the y-coordinate of the point on the graph that is the lowest along the vertical or y-axis. Since the grid lines are spaced 1 unit apart on each axis, the lowest point along the y-axis has coordinates $(-3, -2)$. Therefore, the value of x at the minimum of $f(x)$ is -3.

Choice A is incorrect; -5 is the smallest value for an x-coordinate of a point on the graph of f, not the lowest point on the graph of f. Choice C is incorrect; it is the minimum value of f, not the value of x that corresponds to the minimum of f. Choice D is incorrect; it is the value of x at the maximum value of f, not at the minimum value of f.