

## 1.2 Functions and Their Properties

## Vertical and Horizontal Asymptotes

Target 1A: Analyze functions using specific properties

## Review of Prior Concepts

Identify any discontinuities for  $f(x) = \frac{x^2+7x+10}{x^2-4x-12}$  and describe the type of discontinuity.

$$f(x) = \frac{(x+5)(x+2)}{(x-6)(x+2)}$$

$$= \frac{(x+5)\cancel{(x+2)}}{(x-6)\cancel{(x+2)}}$$

↑ nonremovable @  $x=6$   
 ↘ removable discontin. @  $x=-2$

## More Practice

## Discontinuities

<http://www.ck12.org/Analysis/Discrete-and-Continuous-Functions/lesson/Continuity-and-Discontinuity-PCALC/>

<https://www.youtube.com/watch?v=2n5VzMFJQVY>

## Vertical &amp; Horizontal Asymptotes

## RECALL:

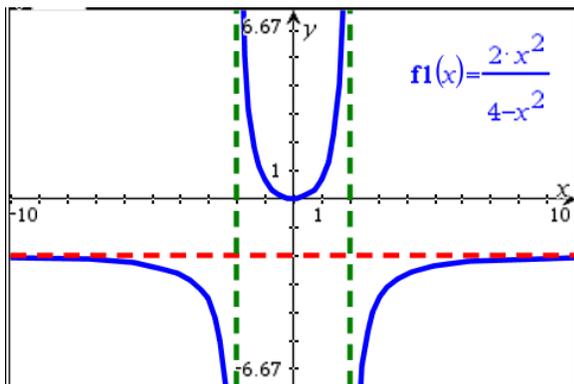
**Vertical Asymptotes** – non-removable discontinuity found from denominator set equal to zero (after common factors have been removed).

**Horizontal Asymptotes** -- occur when end behavior approaches a #,  $c$ . H.A. is @  $y = c$ .

$$\text{NOTATION: } \lim_{x \rightarrow \infty} f(x) = c \quad \text{or} \quad \lim_{x \rightarrow -\infty} f(x) = c$$

Graph each function. Find vertical asymptotes algebraically & horizontal asymptotes graphically (if any).

Example 1:  $g(x) = \frac{2x^2}{4-x^2}$



$$g(x) = \frac{2x^2}{(2-x)(2+x)}$$

$$2-x=0 \quad 2+x=0$$

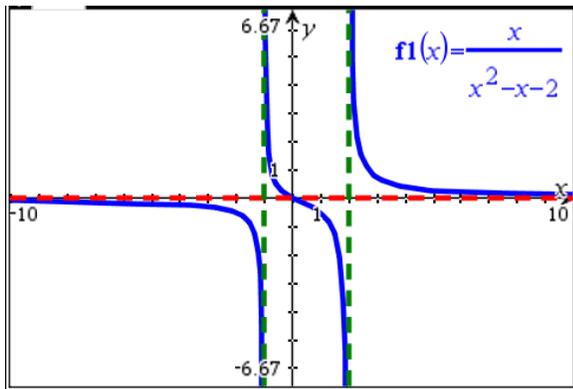
$$2=x \quad x=-2$$

V.A. @  $x=-2, x=2$

H.A. @  $y = -2$  b/c

$$\lim_{x \rightarrow \infty} g(x) = -2, \quad \lim_{x \rightarrow -\infty} g(x) = -2$$

Example 2:  $f(x) = \frac{x}{x^2-x-2}$



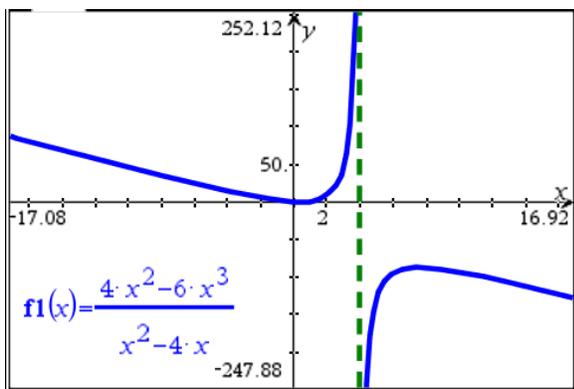
$$f(x) = \frac{x}{(x-2)(x+1)}$$

$$x-2=0 \quad x+1=0$$

$$x=2 \quad x=-1$$
 V.A. @  $x=-1, x=2$

H.A. @  $y=0$  b/c  
 $\lim_{x \rightarrow \infty} f(x) = 0, \lim_{x \rightarrow -\infty} f(x) = 0$

Example 3:  $h(x) = \frac{4x^2-6x^3}{x^2-4x}$



$$h(x) = \frac{x^2(4x-6)}{x(x-4)}$$

$$= \frac{x(x)(4x-6)}{x(x-4)}$$

$$= \frac{x(4x-6)}{x-4}$$

$$x-4=0$$

$$x=4$$
 V.A. @  $x=4$   
 @ Hole  $x=0$

No H.A. b/c  
 $\lim_{x \rightarrow \infty} f(x) = -\infty, \lim_{x \rightarrow -\infty} f(x) = \infty$

**More Practice**

**Vertical Asymptotes**

<http://www.purplemath.com/modules/asymtote.htm>

<https://www.youtube.com/watch?v=h910Jbhzecl>