

## 1.4 Building Functions from Functions

Target 1B: Build functions from functions (composition &amp; inverse)

## Review of Prior Concepts

If  $f(x) = x^2 + 3$  and  $g(x) = x - 1$ , state the domain of  $f(x)$  and  $g(x)$ . Then, find and state the domain of each:

Domain of  $f(x)$ :  $(-\infty, \infty)$   
 Domain of  $g(x)$ :  $(-\infty, \infty)$

**(f+g)(x)**  
 $(f+g)(x) = f(x) + g(x)$   
 $= x^2 + 3 + x - 1$  \*simplify  
 $(f+g)(x) = x^2 + x + 2$   
 Domain:  $(-\infty, \infty)$

**(f-g)(x)**  
 $(f-g)(x) = f(x) - g(x)$  \*make sure to distribute the negative  
 $= x^2 + 3 - (x - 1)$   
 $= x^2 + 3 - x + 1$   
 $(f-g)(x) = x^2 - x + 4$   
 Domain:  $(-\infty, \infty)$

**(fg)(x)**  
 $(fg)(x) = f(x)g(x)$  \*distribute + simplify  
 $= (x^2 + 3)(x - 1)$   
 $(fg)(x) = x^3 - x^2 + 3x - 3$   
 Domain:  $(-\infty, \infty)$

**(f/g)(x)**  
 $(f/g)(x) = \frac{f(x)}{g(x)}$   
 $(f/g)(x) = \frac{x^2 + 3}{x - 1}$   
 not in domain  
 $x - 1 = 0$   
 $x = 1$   
 if you could, you would factor + reduce in this example  
 Domain:  $(-\infty, 1) \cup (1, \infty)$

## More Practice

## Operations on Functions

<https://www.youtube.com/watch?v=z8T-QeTVDuQ>

<https://www.khanacademy.org/math/algebra2/manipulating-functions/combining-functions/a/introduction-to-combining-functions>

[http://www.algebra-lab.org/lessons/lesson.aspx?file=Algebra\\_FunctionsRelationsOperations.xml](http://www.algebra-lab.org/lessons/lesson.aspx?file=Algebra_FunctionsRelationsOperations.xml)

<https://www.mathsisfun.com/sets/functions-operations.html>

**SAT Connection****Passport to Advanced Math**

14. Use structure to isolate or identify a quantity of interest

Example:

$$a = 1,052 + 1.08t$$

The speed of a sound wave in air depends on the air temperature. The formula above shows the relationship between  $a$ , the speed of a sound wave, in feet per second, and  $t$ , the air temperature, in degrees Fahrenheit ( $^{\circ}\text{F}$ ).

Which of the following expresses the air temperature in terms of the speed of a sound wave?

A)  $t = \frac{a - 1,052}{1.08}$

B)  $t = \frac{a + 1,052}{1.08}$

C)  $t = \frac{1,052 - a}{1.08}$

D)  $t = \frac{1.08}{a + 1,052}$

$$a = 1052 + 1.08t$$

$$a - 1052 = 1.08t$$

$$\frac{a - 1052}{1.08} = t$$

Solution**Operations on Functions**

- Sum  $(f + g)(x) = f(x) + g(x)$
- Difference  $(f - g)(x) = f(x) - g(x)$
- Product  $(fg)(x) = f(x)g(x)$
- Quotient  $(f/g)(x) = \frac{f(x)}{g(x)}$  where  $g(x) \neq 0$
- Composition  $(f \circ g)(x) = f(g(x))$   
 $(g \circ f)(x) = g(f(x))$

"f of g of x"  
 "g of f of x" ☺



Example 1:

If  $f(x) = x^2 + 3$  and  $g(x) = x - 1$ , find and state the domain of  $(f \circ g)(x)$  and  $(g \circ f)(x)$ . Then, find  $(f \circ g)(-5)$ .

Handwritten work for Example 1:

Left side:  $(f \circ g)(x) = f(g(x)) = (x-1)^2 + 3 = (x-1)(x-1) + 3 = x^2 - 2x + 1 + 3 = x^2 - 2x + 4$ . Domain:  $(-\infty, \infty)$ . Annotations:  $f(\frac{x}{g(x)}) = \frac{x^2}{g(x)} + 3$ .

Right side:  $(g \circ f)(x) = g(f(x)) = x^2 + 3 - 1 = x^2 + 2$ . Domain:  $(-\infty, \infty)$ . Annotations:  $g(\frac{x}{f(x)}) = \frac{x}{f(x)} - 1$ .

$$(f \circ g)(-5) = f(g(-5)) = (-5)^2 - 2(-5) + 4 = 25 + 10 + 4 = 39$$

Example 2:

If  $f(x) = \sqrt{3-x}$  and  $g(x) = 2x + 8$ , state the domain of  $f(x)$  and  $g(x)$ . Then, find and state the domain of  $(f \circ g)(x)$  and  $(g \circ f)(x)$ . Finally, find  $g(f(-6))$ .

Handwritten work for Example 2:

Domain of  $f(x)$ :  $(-\infty, 3]$ . Note:  $3-x \geq 0 \Rightarrow 3 \geq x \Rightarrow x \leq 3$ . Note:  $3-x \geq 0$  can't have  $\sqrt{\text{negative \#}}$ .

Range of  $g(x)$ :  $(-\infty, \infty)$ .

Left side:  $(f \circ g)(x) = f(g(x)) = \sqrt{3-(2x+8)} = \sqrt{3-2x-8} = \sqrt{-2x-5}$ . Domain:  $(-\infty, -\frac{5}{2}]$ . Note:  $-2x-5 \geq 0 \Rightarrow -2x \geq 5 \Rightarrow x \leq -\frac{5}{2}$ .

Right side:  $(g \circ f)(x) = g(f(x)) = 2\sqrt{3-x} + 8$ . Domain:  $(-\infty, 3]$ .

$$(g \circ f)(-6) = g(f(-6)) = 2\sqrt{3-(-6)} + 8 = 2\sqrt{3+6} + 8 = 2\sqrt{9} + 8 = 2(3) + 8 = 14$$

Example 3:

If  $f(g(x)) = (x+2)^5$ , find the two functions,  $f(x)$  and  $g(x)$ .

Let  $g(x) = y \rightarrow \begin{cases} g(x) = x+2 \\ f(x) = x^5 \end{cases}$   
 $f(y) = y^5 \rightarrow$

**More Practice**

**Composition Functions**

<https://www.youtube.com/watch?v=S4AEZEITPD0>

<https://www.youtube.com/watch?v=kAqaPxusaDg>

<https://www.youtube.com/watch?v=zy7Uro7iCg>

<https://www.youtube.com/watch?v=VsNaX9s1nas>

<https://www.khanacademy.org/math/algebra2/manipulating-functions/function-composition/v/function-composition>

<http://home.windstream.net/okrebs/page42.html>

<https://www.math10.com/en/algebra/functions/operations-on-functions.html>

**Homework Assignment**

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**SAT Connection****Solution**

**Choice A is correct.** Subtracting 1,052 from both sides of the equation  $a = 1,052 + 1.08t$  gives  $a - 1,052 = 1.08t$ . Then dividing both sides of  $a - 1,052 = 1.08t$  by 1.08 gives  $t = \frac{a - 1,052}{1.08}$ .

Choices B, C, and D are incorrect and could arise from errors in rewriting  $a = 1,052 + 1.08t$ . For example, choice B could result if 1,052 is added to the left side of  $a = 1,052 + 1.08t$  and subtracted from the right side, and then both sides are divided by 1.08.