

1.5 Relations and Inverses

Target 1C: Build functions from functions (composition & inverse)

Review of Prior Concepts

Solve for y .

1. $x = 3y - 6$

$$x = 3y - 6$$

$$x + 6 = 3y$$

$$\frac{x+6}{3} = y$$

$$\underline{\underline{y = \frac{x}{3} + 2}}$$

2. $x = y^2 + 4$

$$x - 4 = y^2$$

$$\sqrt{x-4} = \sqrt{y^2}$$

$$\pm \sqrt{x-4} = y$$

3. $x = \frac{y-2}{y+3} \cdot (y+3)$

$$x(y+3) = y-2$$

$$xy + 3x = y - 2$$

$$xy - y = -3x - 2$$

$$y(x-1) = -3x - 2$$

$$\underline{\underline{y = \frac{-3x-2}{x-1}}}$$

More Practice

Solving Equations for a Variable

<http://www.virtualnerd.com/algebra-1/linear-equations-solve/isolate-variables-formulas-examples/isolate-variable/isolate-variables-in-terms-of-variables><http://tutorial.math.lamar.edu/Classes/Alg/SolveMultiVariable.aspx><http://www.youtube.com/watch?v=bjJeyedQLIQ>

SAT Connection

Passport to Advanced Math

14. Use structure to isolate or identify a quantity of interest

Example:

A function f satisfies $f(2) = 3$ and $f(3) = 5$. Afunction g satisfies $g(3) = 2$ and $g(5) = 6$. What is the value of $f(g(3))$?

$$f(g(3)) = f(2) = 3$$

A) 2

B) 3

C) 5

D) 6

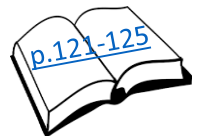
Solution

Inverses Numerically

- An ordered pair (a, b) is in a relation if and only if the ordered pair (b, a) is in the **inverse** relation.

Example 1: Find the inverse of $f(x) = \{(1,3), (2,5), (-4,2), (7,0)\}$

$$f^{-1}(x) = \{(3,1), (5,2), (2,-4), (0,7)\}$$

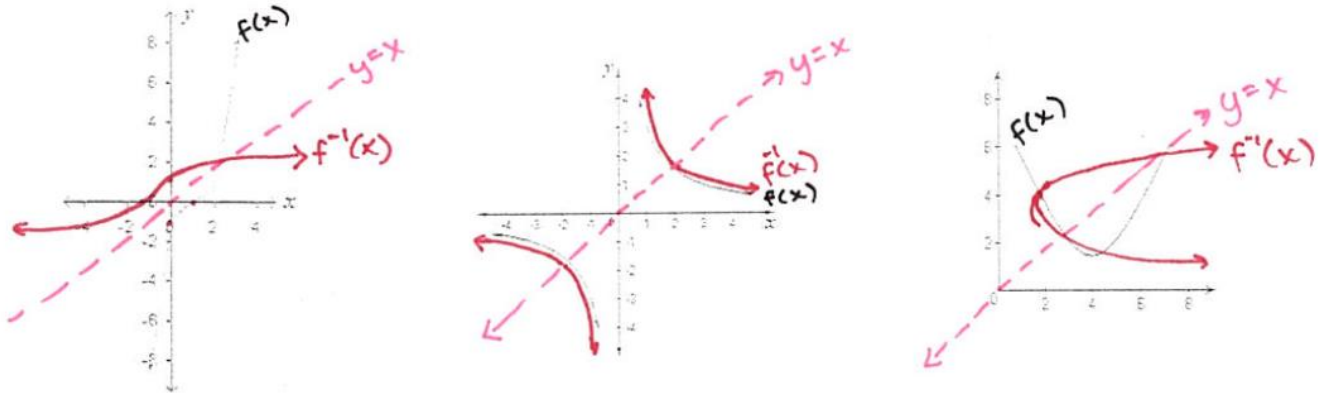


Inverses Graphically

The inverse is a reflection of the function across the line $y = x$.

The notation for the inverse of a function, $f(x)$, is $f^{-1}(x)$.

Example 2: Draw the inverse as a reflection across the line $y = x$.



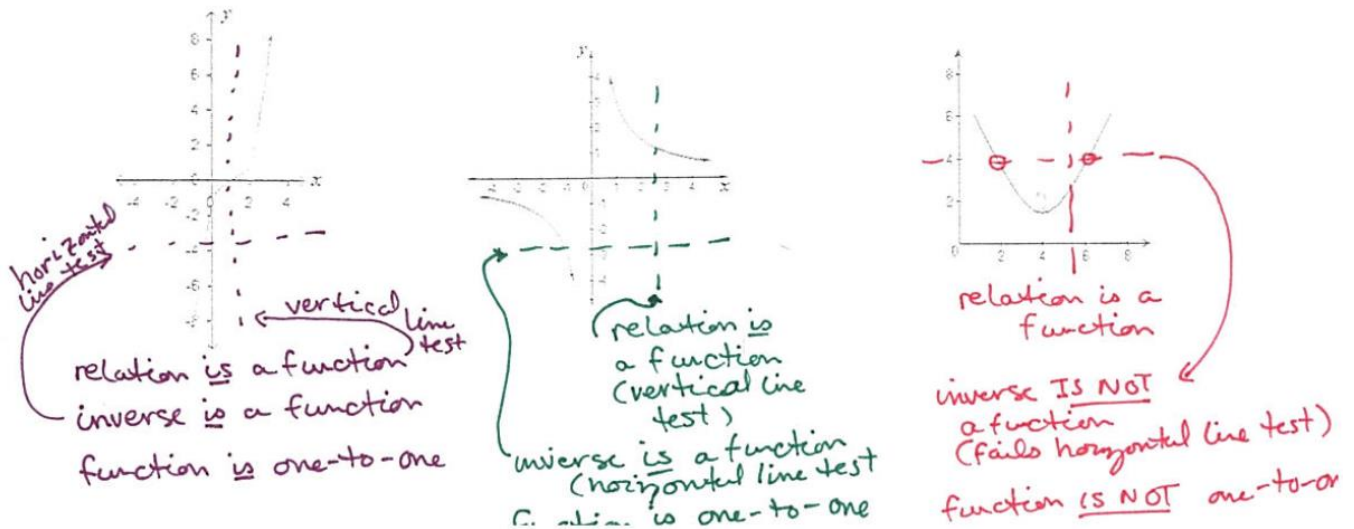
RECALL:

A relation is a function if it passes

the vertical line test

- The inverse of a relation is a function if the relation passes the horizontal line test
- If original relation and inverse are both functions, then the function is called one-to-one

Example 3: Is the relation a function? Is the inverse a function? Is the function one-to-one?



Inverses Algebraically

- 1) Determine that $f(x)$ is one-to-one (or put any restrictions on the domain).
- 2) Switch x & y .
- 3) Solve for y and write with $f^{-1}(x)$ notation.

Examples:

Find the inverse of each function.

1) $f(x) = 3x + 2$

$$x = 3y + 2$$

$$x - 2 = 3y$$

$$\frac{x-2}{3} = y$$

$$f^{-1}(x) = \frac{x-2}{3}$$

2) $g(x) = \frac{x-1}{x+2}$

$$x = \frac{y-1}{y+2}$$

$$x(y+2) = y-1$$

$$xy + 2x = y - 1$$

$$xy - y = -2x - 1$$

$$y(x-1) = -2x-1$$

$$y = \frac{-2x-1}{x-1}$$

$$g^{-1}(x) = \frac{-2x-1}{x-1}$$

3) $h(x) = x^3 - 1$

$$x = y^3 - 1$$

$$x+1 = y^3$$

$$\sqrt[3]{x+1} = y$$

$$h^{-1}(x) = \sqrt[3]{x+1}$$

Determine if $f(x)$ and $g(x)$ are inverses. Show that $f(g(x)) = g(f(x)) = x$.

4) $f(x) = 5x - 2$, $g(x) = \frac{x+2}{5}$

$$f(g(x)) = 5\left(\frac{x+2}{5}\right) - 2$$

$$= x + 2 - 2$$

$$= x \quad \checkmark$$

$$g(f(x)) = \frac{5x-2+2}{5}$$

$$= \frac{5x}{5}$$

$$= x \quad \checkmark$$

 \therefore , $f(x)$ and $g(x)$ are inverses

5) $f(x) = 2x + 1$, $g(x) = \frac{x}{2} - 1$

$$f(g(x)) = 2\left(\frac{x}{2}\right) - 1 + 1$$

$$= x - 2 + 1$$

$$= x - 1$$

$$\neq x \quad \ddot{\cdot}$$

 \therefore , $f(x)$ and $g(x)$ are NOT inverses

6) $f(x) = \frac{x-1}{2x}$, $g(x) = -\frac{1}{2x-1}$

$$f(g(x)) = \frac{-\frac{1}{2x-1} - 1}{2\left(\frac{-1}{2x-1}\right)}$$

$$= \frac{-\frac{1}{2x-1} - \frac{2x-1}{2x-1}}{\frac{-2}{2x-1}}$$

$$= \frac{-\frac{2x}{2x-1}}{\frac{-2}{2x-1}} = \frac{-2x}{2x-1} \cdot \frac{2x-1}{-2} = \frac{-2x(2x-1)}{-2(2x-1)} = x \quad \checkmark$$

$$g(f(x)) = \frac{1}{2\left(\frac{x-1}{2x}\right) - 1}$$

$$= \frac{1}{\frac{x-1}{x} - 1}$$

$$= \frac{1}{\frac{x-1}{x} - \frac{x}{x}}$$

$$= \frac{1}{\frac{-1}{x}} = -1 \cdot \frac{x}{-1} = x \quad \checkmark$$

 \therefore , $f(x)$ and $g(x)$ are inverses

More Practice**Inverse Functions**

<http://www.regentsprep.org/regents/math/algtrig/atp8/inverselesson.htm>

<http://tutorial.math.lamar.edu/Classes/CalcI/InverseFunctions.aspx>

<http://www.mathcentre.ac.uk/resources/uploaded/mc-ty-inverse-2009-1.pdf>

<https://www.youtube.com/watch?v=gXIRspXL6oc>

<https://www.youtube.com/watch?v=JPPitlVKjWQ>

Homework Assignment

p.126 #9,10,13,16,11,23,26,27,31

SAT Connection**Solution**

Choice B is correct. It is given that $g(3) = 2$. Therefore, to find the value of $f(g(3))$, substitute 2 for $g(3)$: $f(g(3)) = f(2) = 3$.

Choices A, C, and D are incorrect and may result from misunderstandings about function notation.