11.1 Limits and Motion: The Tangent Problem

Target 9E: Use and apply the limit definition of the derivative

Review of Prior Concepts

If $f(x) = 2x^2 - 2$, find

a) $f(-1) = 2(-1)^2 - 2$
$= 2 - 2$
$= \boxed{0}$

b) $f(a) = 2(a)^2 - 2$
$= \boxed{2a^2 - 2}$

c) $f(x+h) = 2(x+h)^2 - 2$
$= 2(x^2 + 2xh + h^2) - 2$
$= \boxed{2x^2 + 4xh + 2h^2 - 2}$

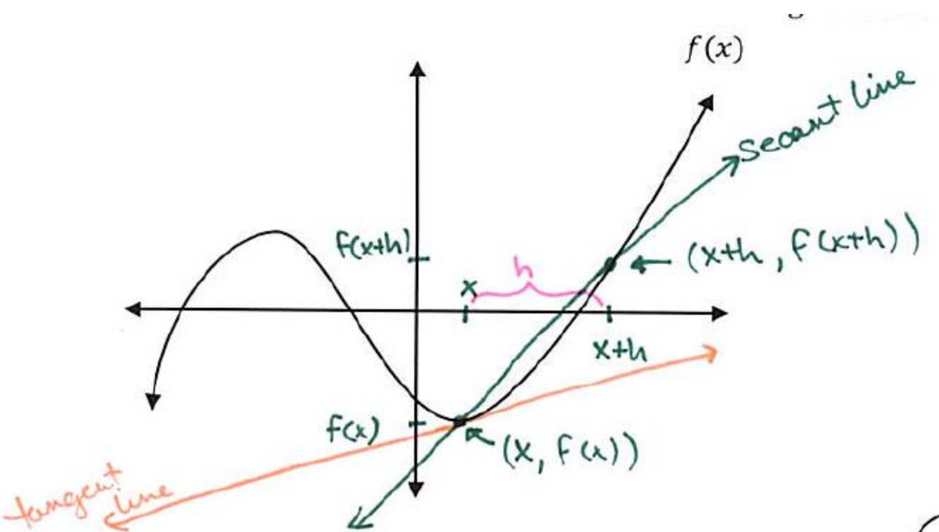
$(x+h)^2 =$
$= (x+h)(x+h)$
$= x^2 + xh + xh + h^2$

d) $\dfrac{f(x+h) - f(x)}{h} = \dfrac{2(x+h)^2 - 2 - (2x^2 - 2)}{h}$
$= \dfrac{2(x^2 + 2xh + h^2) - 2 - 2x^2 + 2}{h}$
$= \dfrac{\cancel{2x^2} + 4xh + 2h^2 \cancel{-2} - \cancel{2x^2} + \cancel{2}}{h}$
$= \dfrac{4xh + 2h^2}{h}$
$= \dfrac{\cancel{h}(4x+2h)}{\cancel{h}}$
$= 4x + 2h$

More Practice

Evaluating Functions
http://www.mathsisfun.com/algebra/functions-evaluating.html
https://www.khanacademy.org/math/algebra/algebra-functions/evaluating-functions/e/functions_1
http://www.coolmath.com/algebra/15-functions/08-the-difference-quotient-01
https://youtu.be/E9YEUQR9NAU
https://youtu.be/1O5NEI8UuHM

The Tangent Problem



secant line - intersects graph @ 2 pts

$m_{\text{secant line}} = \dfrac{y_2 - y_1}{x_2 - x_1}$
$= \dfrac{f(x+h) - f(x)}{x+h-x}$
$= \dfrac{f(x+h) - f(x)}{h}$

Slope of secant line
= average rate of change
= average velocity

tangent line – touches graph @ one point

– moves the pt. $(x+h, f(x+h))$ really close to $(x, f(x))$

→ so, h gets close to zero

$$m_{\text{tangent line}} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Say as: "f prime of x"

slope of tangent line
= instantaneous rate of change
= instantaneous velocity
= $f'(x)$ = derivative

Examples:

1. Find the $\underbrace{\text{instantaneous rate of change}}_{f'(x)}$ @ $x = 2$ for $f(x) = 3x^2 - 2x + 1$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\overbrace{3(x+h)^2 - 2(x+h) + 1}^{f(x+h)} - \overbrace{(3x^2 - 2x + 1)}^{f(x)}}{h}$$

$$= \lim_{h \to 0} \frac{3(x^2 + 2xh + h^2) - 2x - 2h + 1 - 3x^2 + 2x - 1}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 - \cancel{2x} - 2h + \cancel{1} - \cancel{3x^2} + \cancel{2x} - \cancel{1}}{h}$$

$$= \lim_{h \to 0} \frac{6xh + 3h^2 - 2h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(6x + 3h - 2)}{\cancel{h}}$$

$$f'(x) = \lim_{h \to 0} (6x + 3h - 2)$$

$$= 6x - 2$$

$$f'(2) = 6(2) - 2$$

$$= 10$$

$\boxed{\text{instantaneous rate of change @ } x = 2 \text{ is } 10}$

2. Find the $\underbrace{\text{derivative}}_{f'(x)}$ of $f(x) = 8x - 4$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\overbrace{8(x+h) - 4}^{f(x+h)} - \overbrace{(8x - 4)}^{f(x)}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{8x} + 8h - \cancel{4} - \cancel{8x} + \cancel{4}}{h}$$

$$= \lim_{h \to 0} \frac{8\cancel{h}}{\cancel{h}}$$

$$= \lim_{h \to 0} 8$$

$$\boxed{f'(x) = 8}$$

3. Find $f'(x)$ for $f(x) = \sqrt{x}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

conjugate... ☺

$$= \lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$= \lim_{h\to 0} \frac{\cancel{x}+h \cancel{- x}}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$f'(x) = \frac{1}{\sqrt{x+0} + \sqrt{x}}$$

$$\boxed{f'(x) = \frac{1}{2\sqrt{x}}}$$

or $\frac{1}{2}x^{-1/2}$

More Practice

The Tangent Line

http://clas.sa.ucsb.edu/staff/lee/secant,%20tangent,%20and%20derivatives.htm
http://tutorial.math.lamar.edu/Classes/CalcI/Tangents_Rates.aspx
https://youtu.be/qPOUPXlfEWU
https://youtu.be/uI9QLZGqV1A
https://youtu.be/ydHzk5zWd4I

Homework Assignment

p.762 #3,7,9,11,23,25,29
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