Definition of the Derivative

Find $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ for the following. Check your answers by using the "power rule."

1. $f(x)=3x-7$

2. $f(x)=2x+11$

3. $f(x)=-4x+2$

4. $f(x)=-x+6$

5. $f(x)=7x+32$

6. $f(x)=x^2$

7. $f(x)=x^2+3x$

8. $f(x)=3x^2$

9. $f(x)=4x^2$

10. $f(x)=x^3$

11. $f(x)=\frac{1}{x}$

12. $f(x)=\sqrt{x}$

Definition of the Derivative: Key

1. $\boxed{f(x)=3x-7}$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\frac{3(x+h)-7-(3x-7)}{h}=\frac{3x+3h-7-3x+7}{h}=\frac{3h}{h}=\boxed{3}$$

2. $\boxed{f(x)=2x+11}$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\frac{2(x+h)+11-(2x+11)}{h}=\frac{2x+2h+11-2x-11}{h}=\frac{2h}{h}=\boxed{2}$$

3. $\boxed{f(x)=-4x+2}$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\frac{-4(x+h)+2-(-4x+2)}{h}=\frac{-4x-4h+2+4x-2}{h}=\frac{-4h}{h}=\boxed{-4}$$

4. $\boxed{f(x)=-x+6}$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\frac{-(x+h)+6-(-x+6)}{h}=\frac{-x-h+6+x-6}{h}=\frac{-h}{h}=\boxed{-1}$$

5. $\boxed{f(x)=7x+32}$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\frac{7(x+h)+32-(7x+32)}{h}=\frac{7x+7h+32-7x-32}{h}=\frac{7h}{h}=\boxed{7}$$

6. $\boxed{f(x)=x^2}$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\frac{(x+h)^2-x^2}{h}=\frac{x^2+2xh+h^2-x^2}{h}=\frac{2xh+h^2}{h}=2x+h=\boxed{2x}$$

7. $\boxed{f(x)=x^2+3x}$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\frac{(x+h)^2+3(x+h)-(x^2+3x)}{h}=\frac{x^2+2xh+h^2+3x+3h-x^2-3x}{h}=\frac{2xh+h^2+3h}{h}=2x+h+3=\boxed{2x+3}$$

8. $\boxed{f(x)=3x^2}$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\frac{3(x+h)^2-(3x^2)}{h}=\frac{3(x^2+2xh+h^2)-3x^2}{h}=\frac{3x^2+6xh+3h^2-3x^2}{h}=\frac{6xh+3h^2}{h}=6x+3h=\boxed{6x}$$

9. $\boxed{f(x)=4x^2}$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\frac{4(x+h)^2-(4x^2)}{h}=\frac{4(x^2+2xh+h^2)-4x^2}{h}=\frac{4x^2+8xh+4h^2-4x^2}{h}=\frac{8xh+4h^2}{h}=8x+4h=\boxed{8x}$$

10. $\boxed{f(x)=x^3}$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\frac{(x+h)^3-(x^3)}{h}=\frac{x^3+3x^2h+3xh^2+h^3-x^3}{h}=\frac{3x^2h+3xh^2+h^3}{h}=3x^2+3xh+h^2=\boxed{3x^2}$$

11. $\boxed{f(x)=\frac{1}{x}}$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\frac{\frac{1}{x+h}-\frac{1}{x}}{h}=\frac{\frac{x}{x(x+h)}-\frac{x+h}{x(x+h)}}{h}=\frac{\frac{x-x-h}{x(x+h)}}{h}=\frac{\frac{-h}{x(x+h)}}{h}=\frac{-h}{x(x+h)}\cdot\frac{1}{h}=\frac{-1}{x(x+h)}=\boxed{-\frac{1}{x^2}}$$

12. $\boxed{f(x)=\sqrt{x}}$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\frac{\sqrt{x+h}-\sqrt{x}}{h}=\frac{\left(\sqrt{x+h}-\sqrt{x}\right)\left(\sqrt{x+h}+\sqrt{x}\right)}{h\left(\sqrt{x+h}+\sqrt{x}\right)}=\frac{x+h-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)}=\frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)}=\frac{1}{\sqrt{x+h}+\sqrt{x}}=\boxed{\frac{1}{2\sqrt{x}}}$$