Unit 9 (Chapter 11): Limits **DATE: _______________**

11.1 Limits and Motion: The Tangent Problem

Target 9E: Use and apply the limit definition of the derivative

Warm Up

Given $f(x) = x^2 - 1$, find $f'(x)$. Analyze the student's work below. Identify the error(s). Then show how the error(s) can be corrected.

$$f'(x) = \lim_{h \to 0} \frac{x^2 - 1 + h - (x^2 - 1)}{h}$$

$$= \lim_{h \to 0} \frac{x^2 - 1 + h - x^2 + 1}{h}$$

$$= \lim_{h \to 0} \frac{h}{h}$$

$$= \lim_{h \to 0} 1$$

$$= 1$$

Error Analysis Activity

For each problem, analyze the student's work. Identify the error(s). Then show how the error(s) can be corrected.

Find the derivative of $f(x) = 2x + 3$.

$$f'(x) = \lim_{h \to 0} \frac{2(x+h) + 3 - (2x + 3)}{h}$$

$$= \lim_{h \to 0} \frac{2x + h + 3 - 2x + 3}{h}$$

$$= \lim_{h \to 0} 6$$

$$= 6$$

Unit 9 (Chapter 11): Limits

If $f(x) = x^2 - x + 2$, find the derivative of $f(x)$.

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^2 - (x+h) + 2 - (x^2 - x + 2)}{h}$$

$$= \lim_{h\to 0} \frac{x^2 + 2xh - x - h + 2 - x^2 + x - 2}{h}$$

$$= \lim_{h\to 0} \frac{2xh - h}{h}$$

$$= \frac{h(2x-1)}{h}$$

$$= 2x - 1$$

Find $f'(2)$ if $f(x) = \sqrt{x+1}$.

$$f'(x) = \lim_{h\to 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \cdot \frac{\sqrt{x+h+1} + \sqrt{x+1}}{\sqrt{x+h+1} + \sqrt{x+1}}$$

$$= \lim_{h\to 0} \frac{x+h+1 - x+1}{h(\sqrt{x+h+1} + \sqrt{x+1}}$$

$$= \lim_{h\to 0} \frac{2h}{h(\sqrt{x+h+1} + \sqrt{x+1})}$$

$$= \lim_{h\to 0} \frac{2}{\sqrt{x+h+1} + \sqrt{x+1}}$$

$$= \frac{2}{\sqrt{x+0+1} + \sqrt{x+1}}$$

$$= \frac{2}{\sqrt{x+1} + \sqrt{x+1}}$$

$$= \frac{2}{2\sqrt{x+1}}$$

$$= \frac{1}{\sqrt{x+1}}$$