Limits Involving (Approaching) Infinity: $\lim_{x\to\infty} f(x)$

Important Theorem: $\lim_{x\to\infty} \frac{1}{x} = 0$

Limits Involving Infinity (Principle of Dominance)
1. $\lim_{x\to\infty} \frac{x^a}{x^b}$, *if* $a < b$. Then the limit is equal to 0. (Look at the degree of the numerator and denominator, i.e., the greatest power/exponent of x of the polynomial in the numerator and the polynomial in the denominator.)
2. $\lim_{x\to\infty} \frac{Cx^a}{Dx^b}$, *if* $a = b$. Then the limit is a ratio of leading coefficients, $\frac{C}{D}$. (Look at the degree of the numerator and denominator, i.e., the greatest power/exponent of x of the polynomial in the numerator and the polynomial in the denominator.)
3. $\lim_{x\to\infty} \frac{x^a}{x^b}$, *if* $a > b$. Then the limit is equal to ∞ or $-\infty$. (Look at the degree of the numerator and denominator, i.e., the greatest power/exponent of x of the polynomial in the numerator and the polynomial in the denominator. **MUST** check the sign of ∞ by substituting into the rational function a sufficiently large value for every x.)

Directions: Show your work (write out your explanations) on a separate sheet of paper.

Problems:

1. $\lim_{x\to\infty} 7+\frac{1}{3x}-\frac{2}{x^2}$	2. $\lim_{x\to-\infty} \frac{4x+8}{5x}$	3. $\lim_{x\to\infty} \frac{3x-1000}{x+100}$	4. $\lim_{x\to-\infty} \frac{5x+5}{7x^2+1}$
5. $\lim_{x\to\infty} \frac{5x^2+2}{4x^2+7}$	6. $\lim_{x\to-\infty} \frac{3x^3+5}{5x^2+1}$	7. $\lim_{x\to\infty} \frac{2x^2-4x}{x+1}$	8. $\lim_{x\to-\infty} \frac{2x^2-4x}{x+1}$
9. $\lim_{x\to\infty} \frac{3x^3+2}{5x^2-1}$	10. $\lim_{x\to-\infty} \frac{3x^2+2}{4x^2-1}$	11. $\lim_{x\to\infty} \frac{x^2+2}{x-555}$	12. $\lim_{x\to-\infty} \frac{3-2x}{3x^3-1}$
13. $\lim_{x\to\infty} \frac{3-5x}{3x-1}$	14. $\lim_{x\to\infty} \frac{3-2x^2}{3x-1}$	15. $\lim_{x\to\infty} \frac{6x^2-2x-1}{2x^2+3x+2}$	16. $\lim_{x\to\infty} \frac{3x^3+2}{2x^2-9x^3+7}$
17. $\lim_{x\to-\infty} \frac{x}{x^2-1}$	18. $\lim_{x\to-\infty} \frac{8x^2+3x}{2x^2-1}$	19. $\lim_{x\to\infty} 10-\frac{2}{x^2}$	20. $\lim_{x\to-\infty} 4+\frac{3}{x}$
21. $\lim_{x\to-\infty} \frac{5x^2}{x+3}$	22. $\lim_{x\to\infty} \frac{1}{2}x-\frac{4}{x^2}$	23. $\lim_{x\to\infty} \frac{\sin x}{x}$	24. $\lim_{x\to\infty} \frac{\cos 2x}{3x}$

Answers:

1. 7	2. $\frac{4}{5}$	3. 3	4. 0	5. $\frac{5}{4}$	6. $-\infty$
7. ∞	8. $-\infty$	9. ∞	10. $\frac{3}{4}$	11. ∞	12. 0
13. $-\frac{5}{3}$	14. $-\infty$	15. 3	16. $\frac{-1}{3}$	17. 0	18. 4
19. 10	20. 4	21. $-\infty$	22. ∞	23. 0	24. 0