

11.3 More on Limits

Target 9A: Evaluate a limit of a function algebraically
Target 9C: Evaluate a limit of a function graphically

Review of Prior Concepts

HINT: Think about the parent functions

1. $\lim_{x \rightarrow \infty} (x^2 + 3x - 2) = \infty$



2. $\lim_{x \rightarrow -\infty} (x^2 + 3x - 2) = \infty$

3. $\lim_{x \rightarrow \infty} (x^3 + 2) = \infty$



4. $\lim_{x \rightarrow -\infty} (x^3 + 2) = -\infty$

5. $\lim_{x \rightarrow \infty} (e^x) = \infty$



6. $\lim_{x \rightarrow -\infty} (e^x) = 0$

More Practice

End Behavior

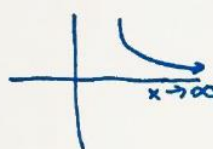
- <http://www.coolmath.com/precalculus-review-calculus-intro/precalculus-algebra/14-tail-behavior-limits-at-infinity-02>
- <https://quizlet.com/48500929/end-behavior-of-12-basic-functions-flash-cards/>
- <http://www.mathguide.com/cgi-bin/quizmasters3/EB.cgi>
- https://youtu.be/Krjd_vU4Uvg

Limits at Infinity

(end behavior of a graph/function)

Example 1:

a) Find $\lim_{x \rightarrow \infty} \left(\frac{3}{x}\right) = 0^+$



positive zero (zero from positive #'s)
graph goes to zero from positive y-values

$\frac{3}{\infty}$

Some big #
 $\frac{3}{10000000} = +0.0000003$

As x approaches ∞ , the graph of $\frac{3}{x}$ approaches 0.

b) Find $\lim_{x \rightarrow -\infty} \left(\frac{3}{x}\right) = 0^-$



graph goes to zero from negative y-values

$\frac{3}{-\infty}$

Some big neg #
 $\frac{3}{-10000000} = -.0000003$

As x approaches $-\infty$, the graph of $\frac{3}{x}$ approaches 0

Example 2:

a) Find $\lim_{x \rightarrow \infty} \left(\frac{x-2}{3x-4} \right)$

$$= \lim_{x \rightarrow \infty} \frac{\frac{1}{x}x - \frac{2}{x}}{\frac{3}{x}x - \frac{4}{x}}$$

$$= \lim_{x \rightarrow \infty} \frac{1 - \frac{2}{x}}{3 - \frac{4}{x}}$$

$$= \boxed{\frac{1}{3}}$$

So, divide by $\frac{x}{x}$ So, do some algebra

divide each term by x \rightarrow variable of leading term

$\frac{2}{x} \rightarrow \frac{2}{\infty} = 0$
 $\frac{4}{x} \rightarrow \frac{4}{\infty} = 0$

Check answer graphically (graph the function & the answer)

(Notice: as $x \rightarrow \infty$, graph approaches Horizontal Asymptote @ $y = \frac{1}{3}$)

b) Find $\lim_{x \rightarrow -\infty} \left(\frac{6x}{3x+4} \right)$

$$= \lim_{x \rightarrow -\infty} \frac{\frac{6x}{x}}{\frac{3x}{x} + \frac{4}{x}}$$

$$= \lim_{x \rightarrow -\infty} \frac{6}{3 + \frac{4}{x}}$$

$$= \frac{6}{3} = \boxed{2}$$

$\frac{4}{x}$ goes to zero

c) Find $\lim_{x \rightarrow \infty} \left(\frac{8x-1}{2x+3} \right) = \frac{8}{2} = \boxed{4}$

(Can you see the answer quickly—w/o work?)

Conclusion:

If degree of Numerator = degree of Denominator, then $\lim_{x \rightarrow \infty} f(x) = \frac{\text{coefficients of leading terms of Numerator/Denominator}}$
 (and, Horizontal Asymptote @ $y = \text{coefficients (\#)}$)

Example 3:

a) Find $\lim_{x \rightarrow \infty} \left(\frac{x-2}{3x^2-4} \right) = 0$

b) Find $\lim_{x \rightarrow -\infty} \left(\frac{6x}{3x^2+4} \right) = 0$

$\frac{\infty}{\infty}$

$\infty - 2 = \text{big \#}$
 $3(\infty)^2 - 4 = \text{even bigger \#}$
 ex: $\frac{1000}{1000000} = .001$

proof: $\lim_{x \rightarrow \infty} \frac{\frac{x}{x^2} - \frac{2}{x^2}}{\frac{3x^2}{x^2} - \frac{4}{x^2}}$ divide by variable largest leading term

$$= \lim_{x \rightarrow \infty} \frac{\frac{1}{x} - \frac{2}{x^2}}{3 - \frac{4}{x^2}}$$

$$= \frac{0}{3} = 0$$

$\frac{1}{x} \rightarrow 0$
 $\frac{2}{x^2} \rightarrow 0$
 $\frac{4}{x^2} \rightarrow 0$

Conclusion:

If degree of Numerator < degree of Denominator, then $\lim_{x \rightarrow \infty} f(x) = 0$
 (and, Horizontal Asymptote @ $y = 0$)

Example 4:

a) Find $\lim_{x \rightarrow \infty} \left(\frac{x^2 - 2}{3x - 4} \right) = \infty$

$\frac{\infty}{\infty}$ ☹️

$\frac{\infty^2 - 2}{3(\infty) - 4} = \frac{\text{even bigger \#}}{\text{big \#}}$
Ans: $\frac{1000000}{1000} = 1000$
still big #

proof: $\lim_{x \rightarrow \infty} \frac{x^2 - 2}{3x - 4}$ divide by variable of largest leading term

$\lim_{x \rightarrow \infty} \frac{1 - \frac{2}{x^2}}{3 - \frac{4}{x}}$

$\lim_{x \rightarrow \infty} \frac{1 - 0}{3 - 0} = \frac{1}{3} = 0.333333$

b) Find $\lim_{x \rightarrow -\infty} \left(\frac{6x^2}{3x + 4} \right) = -\infty$

☺️ why negative?

$\frac{6(\infty)^2}{3(-\infty) + 4} = \frac{+\infty}{-\infty}$
pos / neg = neg

Conclusion:

If degree of Numerator $>$ degree of Denominator, then $\lim_{x \rightarrow \infty} f(x) = +\infty$ or $-\infty$

(NO Horizontal Asymptote)

More Practice

Limits at Infinity

<https://www.mathsisfun.com/calculus/limits-infinity.html>

<https://www.khanacademy.org/math/ap-calculus-ab/infinite-limits-ab/limits-at-infinity-ab/v/limits-and-infinity>

<http://www.shmoop.com/precalculus-limits/limits-infinity.html>

<https://youtu.be/wBYr-58mc5E>

<https://youtu.be/75xO9xy7TTQ>

<https://youtu.be/FVJNuukADeQ>

Homework Assignment

Infinite Limits Worksheet