

11.3 More on Limits

Target 9C: Evaluate a limit of a function graphically

Target 9D: Calculate one-sided limits and two-sided limits

Review of Prior Concepts

Given $\lim_{x \rightarrow 3} f(x) = 2$ and $\lim_{x \rightarrow 3} g(x) = -1$, find:

a) $\lim_{x \rightarrow 3} (2f(x) + g(x))$

$$= 2 \lim_{x \rightarrow 3} f(x) + \lim_{x \rightarrow 3} g(x)$$

$$= 2 \cdot 2 + -1$$

$$= \boxed{3}$$

b) $\lim_{x \rightarrow 3} \frac{(g(x))^2}{f(x)-4}$

$$= \frac{[\lim_{x \rightarrow 3} g(x)]^2}{\lim_{x \rightarrow 3} f(x) - \lim_{x \rightarrow 3} 4}$$

$$= \frac{(-1)^2}{2 - 4} = \boxed{\frac{1}{-2}}$$

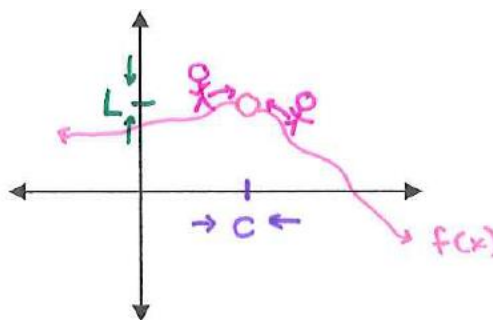
More Practice

Limits Analytically

<http://www.ck12.org/book/CK-12-Precalculus-Concepts/section/14.4/><http://www.ck12.org/book/CK-12-Precalculus-Concepts/section/14.5/><http://precalculus.flippedmath.com/151-limits-analytically.html><http://www.barrington220.org/cms/lib8/IL01001296/Centricity/Domain/321/1.3%20D1%20Ans.pdf><https://youtu.be/-gjURkNuh9o><https://youtu.be/MspCIN-r8C0>

Finding Limits Graphically

$$\lim_{x \rightarrow c} f(x) = L$$

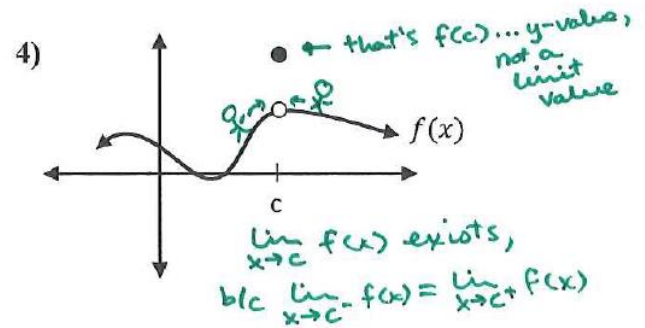
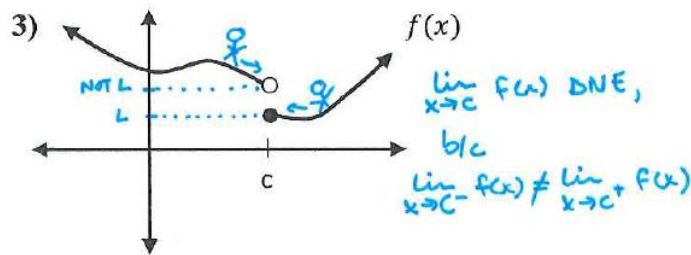
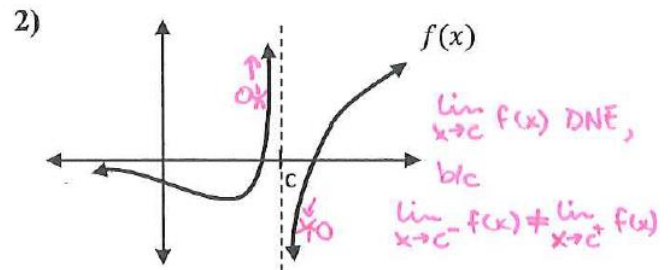
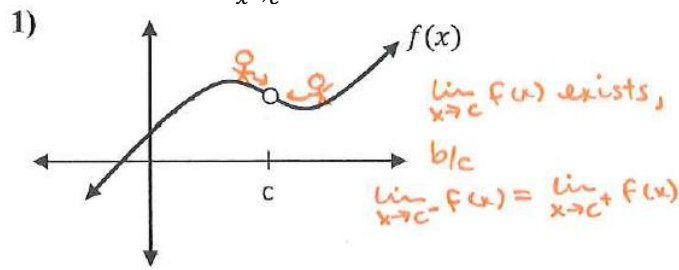
In order for $\lim_{x \rightarrow c} f(x)$ to exist,

$\lim_{x \rightarrow c^-} f(x)$ has to equal $\lim_{x \rightarrow c^+} f(x)$

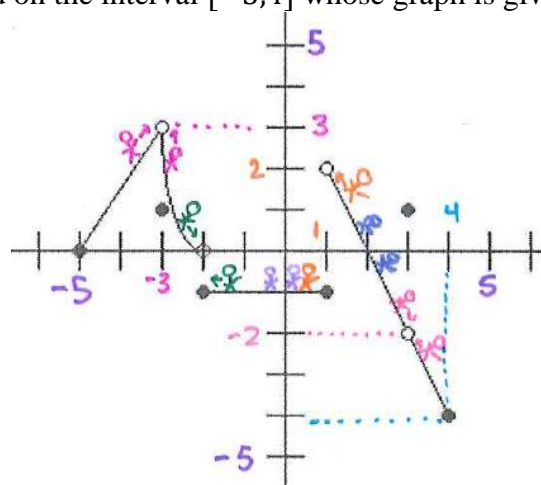
From left From right

"limit from the left = limit from the right" ☺

Examples: Does $\lim_{x \rightarrow c} f(x)$ exist? Explain why or why not.



Let g be a function defined on the interval $[-5, 4]$ whose graph is given as



Using the graph, find the following limits if they exist, and, if not, explain why not.

- 1) $\lim_{x \rightarrow 3} g(x) = -2$
- 2) $\lim_{x \rightarrow 0} g(x) = -1$
- 3) $\lim_{x \rightarrow -3} g(x) = 3$
- 4) $\lim_{x \rightarrow 1^+} g(x) = 2$
- 5) $\lim_{x \rightarrow 1^-} g(x) = -1$
- 6) $\lim_{x \rightarrow 1} g(x) = \text{DNE}$
b/c $\lim_{x \rightarrow 1^-} g(x) \neq \lim_{x \rightarrow 1^+} g(x)$
- 7) $\lim_{x \rightarrow -2^-} g(x) = 0$
- 8) $\lim_{x \rightarrow 4} g(x) = \text{DNE}$
 $\lim_{x \rightarrow 4^-} g(x) = -4$
 $\lim_{x \rightarrow 4^+} g(x) = \text{DNE}$
 $\therefore \lim_{x \rightarrow 4^-} g(x) \neq \lim_{x \rightarrow 4^+} g(x)$
- 9) $\lim_{x \rightarrow 2} g(x) = 0$
- 10) $\lim_{x \rightarrow -2^+} g(x) = -1$

More Practice

Limits Graphically

<http://www.coolmath.com/precaculus-review-calculus-intro/precaculus-algebra/21-rational-functions-limits-infinity-right-left-01>

<https://www.khanacademy.org/math/ap-calculus-ab/limits-basics-ab/limits-from-graphs-ab/v/limits-from-graphs-undefined>

<http://precaculus.flippedmath.com/23-limits-graphically.html>

<http://philschatz.com/precaculus-book/contents/m49452.html>

<http://www.mathsisfun.com/calculus/limits.html>

<https://youtu.be/Ik1VRSonvFQ>

<https://youtu.be/UkjgJQaGx98>

https://youtu.be/XOu_LFGai0A

Homework Assignment

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