

### 11.3 More on Limits

Target 9B: Evaluate a limit of a function numerically  
 Target 9D: Calculate one-sided limits and two-sided limits

Review of Prior Concepts

Graph function  $h(x)$  such that it has all of the characteristics below:

$h(-3) = 0, h(0) = 4$  and  $h(1) = 1$

all pts...  $(-3, 0), (0, 4), (1, 1)$   
 $(3, 1), (5, 2)$

$h(3) = 1$  and  $h(5) = 2$

$\lim_{x \rightarrow 0^+} h(x) = 1$  limit from right not same as y-value, so open circle @  $(0, 1)$

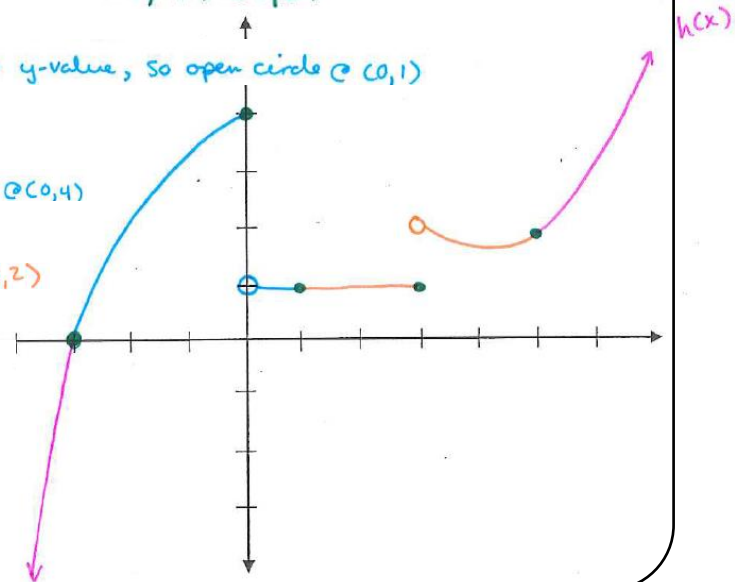
$\lim_{x \rightarrow 0^-} h(x) = 4$   
 limit from left same as y-value, so filled in @  $(0, 4)$

$\lim_{x \rightarrow 3^+} h(x) = 2$   
not same as y-value, so open circle @  $(3, 2)$

$\lim_{x \rightarrow 3^-} h(x) = 1$   
same as y-value, so filled in @  $(3, 1)$

$\lim_{x \rightarrow -\infty} h(x) = -\infty$

$\lim_{x \rightarrow \infty} h(x) = \infty$  } end behavior of graph



#### More Practice

##### Limits Graphically

- <http://www.coolmath.com/precalculus-review-calculus-intro/precalculus-algebra/21-rational-functions-limits-infinity-right-left-01>
- <https://www.khanacademy.org/math/ap-calculus-ab/limits-basics-ab/limits-from-graphs-ab/v/limits-from-graphs-undefined>
- <http://precalculus.flippedmath.com/23-limits-graphically.html>
- <http://philschatz.com/precalculus-book/contents/m49452.html>
- <http://www.mathsisfun.com/calculus/limits.html>
- <https://youtu.be/Ik1VR5onvFQ>
- <https://youtu.be/UkjgJQaGx98>
- [https://youtu.be/XOU\\_LFGai0A](https://youtu.be/XOU_LFGai0A)

#### Finding Limits Numerically

- Use a table of values to find the limit.

Examples:

1. Find  $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$

$x$	2.9	2.99	2.999	3	3.001	3.01	3.1
$f(x)$	5.9	5.99	5.999	undefined	6.001	6.01	6.1

let  $f(x) = \frac{x^2 - 9}{x - 3}$

use graph calc, table of values

$\lim_{x \rightarrow 3^-} f(x) = 6$

$\lim_{x \rightarrow 3^+} f(x) = 6$

$\therefore \lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} = 6$  b/c  
 $\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^+} f(x)$

remember algebra:  $(x-3)(x+3)$   
 $\frac{x^2 - 9}{x - 3}$   
 HOLE @  $x = 3$

# less than 3  
 # greater than 3  
 from left  $\rightarrow 3$  from right  $\leftarrow 3$   
 # closer  
 # even closer  
 # close to 3

2. Find  $\lim_{x \rightarrow 4} \sqrt{x-4}$

$f(x) = \sqrt{x-4}$

x	3.9	3.99	3.999	4	4.001	4.01	4.1
f(x)	DNE	DNE	DNE	0	.032	.1	.316

$\lim_{x \rightarrow 4^-} f(x) = \text{DNE}$        $\lim_{x \rightarrow 4^+} f(x) = 0$

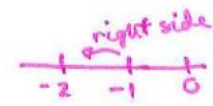
$\lim_{x \rightarrow 4} \sqrt{x-4}$  DNE b/c  $\lim_{x \rightarrow 4^-} f(x) \neq \lim_{x \rightarrow 4^+} f(x)$

3. Find  $\lim_{x \rightarrow -2^+} \frac{1}{x+2}$

$f(x) = \frac{1}{x+2}$

x				-2	-1.999	-1.99	-1.9
f(x)				DNE	1000	100	10

only right side needed



$\lim_{x \rightarrow -2^+} f(x) = \infty$

$\therefore \lim_{x \rightarrow -2} \frac{1}{x+2} = \infty$

4. Find  $\lim_{x \rightarrow 5} \frac{1}{x^2-25}$

$f(x) = \frac{1}{x^2-25}$

x	4.9	4.99	4.999	5	5.001	5.01	5.1
f(x)	-1.01	-10.01	-100.01	DNE	99.99	9.99	.099

$\lim_{x \rightarrow 5^-} f(x) = -\infty$

$\lim_{x \rightarrow 5^+} f(x) = \infty$

$\therefore \lim_{x \rightarrow 5} \frac{1}{x^2-25}$  DNE b/c  $\lim_{x \rightarrow 5^-} f(x) \neq \lim_{x \rightarrow 5^+} f(x)$

Looks like 100... but try #s closer to 5 to see  $\infty$

Limits of Piecewise Functions

Examples:

1.  $f(x) = \begin{cases} 2-x, & \text{if } x < 2 \\ x^2, & \text{if } x \geq 2 \end{cases}$

Find:

a)  $\lim_{x \rightarrow 2^-} f(x)$   
 left... # smaller than 2  
 $= \lim_{x \rightarrow 2^-} (2-x)$   
 $= 2 - (-2)$   
 $= 4$

b)  $\lim_{x \rightarrow 2^+} f(x)$   
 right... # greater than 2  
 $= \lim_{x \rightarrow 2^+} x^2$   
 $= (-2)^2$   
 $= 4$

c)  $\lim_{x \rightarrow 2} f(x)$   
 $\lim_{x \rightarrow 2} f(x) = 4$   
 b/c  $\lim_{x \rightarrow 2} f(x) = \lim_{x \rightarrow 2^+} f(x) = 4$

d)  $f(2)$   
 $f(2) = (-2)^2 = 4$

y-value @  $x=2$

$$2. f(x) = \begin{cases} x - 4, & \text{if } x < 3 \\ 5, & \text{if } x = 3 \\ 8 - x^2, & \text{if } x > 3 \end{cases}$$

*less than 3 ... left of 3*  
*equal to 3*  
*greater than 3 ... right of 3*

Find:

a)  $\lim_{x \rightarrow 3^-} f(x)$

$$= \lim_{x \rightarrow 3^-} (x - 4)$$

$$= 3 - 4$$

$$= \boxed{-1}$$

b)  $\lim_{x \rightarrow 3^+} f(x)$

$$= \lim_{x \rightarrow 3^+} (8 - x^2)$$

$$= 8 - (3)^2$$

$$= 8 - 9$$

$$= \boxed{-1}$$

c)  $\lim_{x \rightarrow 3} f(x)$

$$\lim_{x \rightarrow 3} f(x) = \boxed{-1}$$

b/c

$$\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^+} f(x) = -1$$

d)  $f(3)$

$$f(3) = \boxed{5}$$

### More Practice

#### Limits Numerically

<https://www.khanacademy.org/math/ap-calculus-ab/limits-basics-ab/limits-from-tables-ab/v/limit-by-analyzing-numerical-data>

<https://ww2.coastal.edu/aicogni/Numerical-Limits.pdf>

<http://www.rootmath.org/calculus/estimating-limits-numerically>

<http://goblues.org/faculty/kollathl/files/2010/08/Limits-and-Calculus-slides.pdf>

<https://youtu.be/13a3yGKdED4>

<https://youtu.be/AFIGjj2cWLY>

<https://youtu.be/17Tcay720vw>

#### Piece-Wise Functions

<https://www.khanacademy.org/math/ap-calculus-ab/limits-from-equations-ab/limits-of-piecewise-functions-ab/v/limit-of-piecewise-function-that-is-defined>

<http://www.mathwarehouse.com/calculus/limits/limits-with-piecewise-functions.php>

<https://youtu.be/n0c2DQ00k80>

<https://youtu.be/m-8zaGdm5zA>

<https://youtu.be/Jb09ky2gOVc>

### Homework Assignment

#### Limits Numerically Worksheet