PA 9-5 Limits Numerically

Worksheet
Limits: A Numerical and Graphical Approach

1. Use your graphing calculator to graph $f(x) = \dfrac{\cos x}{x^2}$. Make a guess as to the value of $\lim\limits_{x\to 0} \dfrac{\cos x}{x^2}$. Construct a table of values for $f(-.1), f(-.01), f(-.001), f(-.0001), f(.1), f(.01), f(.001), f(.0001)$. Estimate $\lim\limits_{x\to 0} \dfrac{\cos x}{x^2}$.

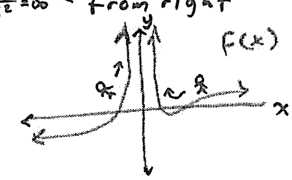
x	-.1	-.01	-.001	-.0001	0	.0001	.001	.01	.1
f(x)	99.5	9999.5	999999.5	99999999.5	DNE or undef.	99999999.5	999999.5	9999.5	99.5

$\lim\limits_{x\to 0^-} \frac{\cos x}{x^2} = \infty$ from left

$\lim\limits_{x\to 0^+} \frac{\cos x}{x^2} = \infty$ from right

As $x \to \infty$, the y-values are getting bigger and bigger, approaching infinity ∵

$$\boxed{\begin{array}{l}\lim\limits_{x\to 0} \dfrac{\cos x}{x^2} = \infty \\ \text{b/c } \lim\limits_{x\to 0^-} \dfrac{\cos x}{x^2} = \lim\limits_{x\to 0^+} \dfrac{\cos x}{x^2}\end{array}}$$



2. Graph $f(x) = x\dfrac{|x-1|}{x-1}$. What is the $\lim\limits_{x\to 1+} f(x)$ and $\lim\limits_{x\to 1-} f(x)$? Construct a table of values for $f(.9), f(.99), f(.999), f(1.001), f(1.01), f(1.1)$. What is the $\lim\limits_{x\to 1+} f(x)$ and $\lim\limits_{x\to 1-} f(x)$?

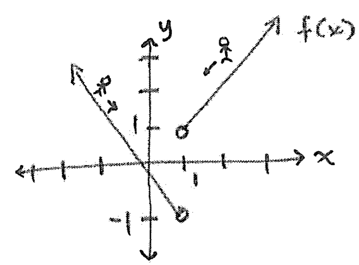
x	.9	.99	.999	1	1.001	1.01	1.1
f(x)	-.9	-.99	-.999	D.N.E. or undefined	1.001	1.01	1.1

from left

from right

$$\boxed{\lim_{x\to 1^-} f(x) = -1}$$

$$\boxed{\lim_{x\to 1^+} f(x) = 1}$$

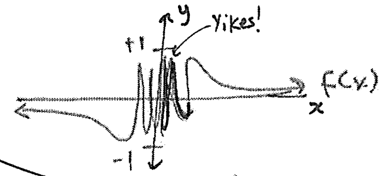


3. Using a graphing calculator, graph $f(x) = \sin\frac{1}{x}$. Does if look as if $\lim_{x\to 0} f(x)$ exists? Construct a table of values for $f(-.1), f(-.01), f(-.001), f(-.0001), f(.1), f(.01), f(.001), f(.0001)$. What do you conclude about $\lim_{x\to 0} f(x)$?

x	−.1	−.01	−.001	−.0001	0	.0001	.001	.01	.1
f(x)	.544	.506	−.827	.306	D.N.E. or undefined	−.306	.827	−.506	−.544

from left ⟶ ⟵ from right

$$\boxed{\lim_{x\to 0} f(x) = \text{D.N.E.}}$$



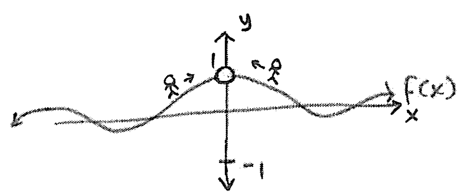
"oscillating"

4. Using a graphing calculator, graph $f(x) = \frac{\sin x}{x}$. Make a guess as to the $\lim_{x\to 0} f(x)$. Construct a table of values for $f(-.1), f(-.01), f(-.001), f(-.0001), f(.1), f(.01), f(.001), f(.0001)$. Estimate $\lim_{x\to 0} f(x)$.

x	−.1	−.01	−.001	−.0001	0	.0001	.001	.01	.1
f(x)	.998	.999	.999	.999	D.N.E. or undefined	.999	.999	.999	.998

from left ⟶ $\lim_{x\to 0^-} f(x) = 1$ $\lim_{x\to 0^+} f(x) = 1$ ⟵ from right

$$\boxed{\lim_{x\to 0} \frac{\sin x}{x} = 1 \quad \text{b/c} \quad \lim_{x\to 0^-} \frac{\sin x}{x} = 1 = \lim_{x\to 0^+} \frac{\sin x}{x}}$$



Recall we already know the above fact (see notes).
Here we confirm numerically.