

Limits: Numerically, Analytically, and Graphically

1. If $f(x) = \frac{\sqrt{x+8}-2}{x+4}$, find $\lim_{x \rightarrow -4} f(x)$ numerically.

x	-4.1	-4.01	-4.001	-4	-3.999	-3.99	-3.9
$f(x)$	0.252	0.250	0.250	Undef. or D.N.E.	0.250	0.250	0.248

$$\lim_{x \rightarrow -4^-} f(x) = 0.25$$

$$\lim_{x \rightarrow -4^+} f(x) = 0.25$$

$$\therefore \lim_{x \rightarrow -4} f(x) = 0.25 \quad \text{b/c} \quad \lim_{x \rightarrow -4^-} f(x) = \lim_{x \rightarrow -4^+} f(x)$$

2. Find $\lim_{x \rightarrow -4} \frac{\sqrt{x+8}-2}{x+4}$ analytically (algebraically).

$$\lim_{x \rightarrow -4} \frac{\sqrt{x+8}-2}{x+4} \cdot \frac{\sqrt{x+8}+2}{\sqrt{x+8}+2} \quad \text{Multiply conjugate}$$

$$= \lim_{x \rightarrow -4} \frac{(\sqrt{x+8})^2 + 2\sqrt{x+8} - 2\sqrt{x+8} - 4}{(x+4)(\sqrt{x+8}+2)} \quad \text{Distribute \& simplify}$$

$$= \lim_{x \rightarrow -4} \frac{x+8-4}{(x+4)(\sqrt{x+8}+2)}$$

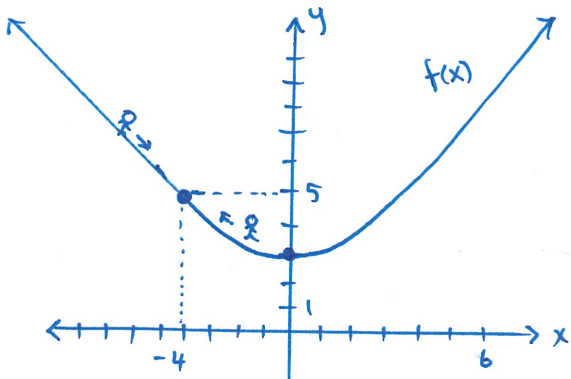
$$= \lim_{x \rightarrow -4} \frac{x+4}{(x+4)(\sqrt{x+8}+2)}$$

$$= \lim_{x \rightarrow -4} \frac{1}{\sqrt{x+8}+2}$$

$$= \frac{1}{\sqrt{-4+8}+2} \quad \text{Replace } -4 \text{ for } x \text{ (don't write limit)}$$

$$= \boxed{\frac{1}{4}}$$

3. If $f(x) = \begin{cases} 1-x & x < -4 \\ 5 & x = -4 \\ \sqrt{x^2+9} & x > -4 \end{cases}$, find $\lim_{x \rightarrow -4} f(x)$ graphically.



$$\lim_{x \rightarrow -4^-} f(x) = 5$$

$$\lim_{x \rightarrow -4^+} f(x) = 5$$

$$\therefore \lim_{x \rightarrow -4} f(x) = 5 \quad \text{b/c} \quad \lim_{x \rightarrow -4^-} f(x) = \lim_{x \rightarrow -4^+} f(x)$$