

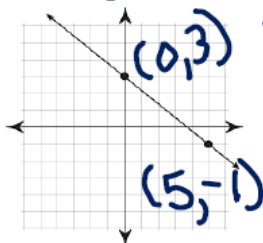
2.1 Linear & Quadratic Functions and Modeling

Review Target: Graph and Solve Quadratic Functions

Review of Prior Concepts

Find the slope of the line:

a)



$$m = \frac{-1-3}{5-0} = \frac{-4}{5}$$

b) that contains the points (2, -3) and (5, 1)

$$m = \frac{1-(-3)}{5-2} = \frac{4}{3}$$

More Practice

Finding Slope of a Line

<http://www.coolmath.com/algebra/08-lines/06-finding-slope-line-given-two-points-01>
<https://www.khanacademy.org/math/algebra/two-var-linear-equations/slope/v/slope-of-a-line>
<http://www.mathwarehouse.com/algebra/linear-equation/slope-of-a-line.php>
https://www.youtube.com/watch?v=Z31F_75C_VE


SAT Connection

Heart of Algebra

1. Create, solve, or interpret a linear expression or equation in one variable that represents a context.

Example:

$$h = 3t + 28.6 \quad m = \frac{\text{change in height}}{\text{change in age}} \Rightarrow \frac{3}{1}$$

A pediatrician uses the model above to estimate the height h of a boy, in inches, in terms of the boy's age a , in years, between the ages of 2 and 5. Based on the model, what is the estimated increase, in inches, of a boy's height each year?

for every 1 year,
height changes
3 inches.

- A) 3
B) 5.7
C) 9.5
D) 14.3

Solution

Polynomial Functions

A polynomial function of degree n (where n is a nonnegative integer) is written as:

$$f(x) = a_m x^m + a_{m-1} x^{m-1} + \dots + a_1 x^1 + a_0$$



Example:

Write a sample polynomial:

$$f(x) = 23x^7 + 20x^6 + 17x^5 + 14x^4 - 10x^3 + 2$$

Degree: 7

Leading coefficient: 23

Name	Form	Degree
Zero Function	$f(x) = 0$	no degree
Constant Function	$f(x) = c$ ← constant	0
Linear Function	$f(x) = ax + b$	1
Quadratic Function	$f(x) = ax^2 + bx + c$	2

Linear Function

$$f(x) = ax + b$$

One of the forms below is needed to write a linear function.

Slope-Intercept Form	Point-Slope Form
$y = mx + b$ <p>↑ slope ↑ y-intercept</p> <p>slope = $\frac{\text{rise}}{\text{run}}$</p>	$y - y_1 = m(x - x_1)$ <p>↓ slope ↑ point</p> <p>2 pts (x_1, y_1) (x_2, y_2)</p> $m = \frac{y_2 - y_1}{x_2 - x_1}$

Example: Write an equation for the linear function, $f(x)$, where $f(1) = 3$ and $f(-2) = 9$.

* Write equation → $y - y_1 = m(x - x_1)$

* Sub in m, x_1, y_1

$$y - 3 = -2(x - 1)$$

$$\begin{array}{r} +3 \qquad \qquad +3 \\ \hline y = -2(x - 1) + 3 \end{array}$$

* Solve for y and write as function → $f(x) = -2(x - 1) + 3$ or $f(x) = -2x + 5$

pt $(1, 3)$ pt $(-2, 9)$

x_1, y_1 x_2, y_2

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{9 - 3}{-2 - 1} = \frac{6}{-3} = -2$$

Quadratic Function

Standard Form	Vertex Form
$f(x) = ax^2 + bx + c$	$f(x) = a(x-h)^2 + k$
vertex: $(-\frac{b}{2a}, f(-\frac{b}{2a}))$	vertex: (h, k)
axis of symmetry: $x = -\frac{b}{2a}$	axis of symmetry: $x = h$
opens up $\rightarrow a > 0$ down $\rightarrow a < 0$	opens up $\rightarrow a > 0$ down $\rightarrow a < 0$

Find the vertex, find the axis of symmetry, and describe the opening of the function:

Example 1:

$$f(x) = 3(x+2)^2 - 7$$

vertex: $(-2, -7)$

axis of symmetry: $x = -2$

opens up b/c $a > 0$

$a = 3 \dots \text{😊}$

Example 2:

$$g(x) = -2x^2 + 7x - 3$$

vertex: $(-\frac{7}{2(-2)}, g(-\frac{7}{2(-2)}))$
 $(1.75, g(1.75))$
 $(1.75, 3.125)$

$$g(1.75) = -2(1.75)^2 + 7(1.75) - 3 = 3.125$$

axis of symmetry: $x = 1.75$

opens down b/c $a < 0$

$a = -2 \dots \text{😊}$

Example 3:

$$h(x) = 8 + 2x - x^2 \rightarrow h(x) = -x^2 + 2x + 8$$

vertex: $(-\frac{2}{2(-1)}, h(-\frac{2}{2(-1)}))$

$(1, h(1))$

$(1, 9)$

$$h(1) = -(1)^2 + 2(1) + 8 = 9$$

axis of symmetry: $x = 1$

opens down b/c $a < 0$

$a = -1 \dots \text{😊}$

SAT Connection**Solution**

Choice A is correct. In the equation $h = 3a + 28.6$, if a , the age of the boy, increases by 1, then h becomes $h = 3(a + 1) + 28.6 = 3a + 3 + 28.6 = (3a + 28.6) + 3$. Therefore, the model estimates that the boy's height increases by 3 inches each year.

Alternatively: The height, h , is a linear function of the age, a , of the boy. The coefficient 3 can be interpreted as the rate of change of the function; in this case, the rate of change can be described as a change of 3 inches in height for every additional year in age.

Choices B, C, and D are incorrect and are likely to result from common errors in calculating the value of h or in calculating the difference between the values of h for different values of a .