DATE: __________

Unit 2 (Chapter 2): Polynomial & Rational Functions **Pre-Calculus**

2.3 Polynomial Functions of Higher Degree w/Modeling

Target 2A: Graph, Solve and Analyze Polynomial Functions

Review of Prior Concepts

Find the degree and leading coefficient of: $f(x) = 5x^2 - 4x^3 + 2 - 7x$.

degree: 3 (highest exponent)

leading coefficient: −4 (L.C.)

End Behavior of polynomials:

→ What happens to the graph as $x \to -\infty$ and $x \to \infty$

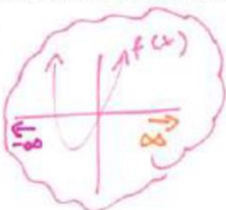
Notation	Meaning of the Notation
$\lim_{x\to-\infty} f(x)$	as x goes to −∞, f(x) approaches some y-value
$\lim_{x\to\infty} f(x)$	as x goes to ∞, f(x) approaches some y-value

Using a graphing calculator, describe the end behavior of the function.

1. $f(x) = x^2 + 3x - 1$

$\lim_{x\to-\infty} f(x) = \infty$

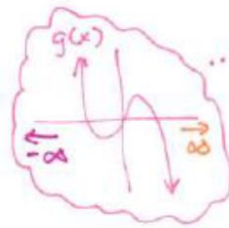
$\lim_{x\to\infty} f(x) = \infty$



2. $g(x) = -x^3 + 2x$

$\lim_{x\to-\infty} f(x) = \infty$

$\lim_{x\to\infty} f(x) = -\infty$



More Practice

End Behavior

http://www.coolmath.com/precalculus-review-calculus-intro/precalculus-algebra/14-tail-behavior-limits-at-infinity-02

https://www.youtube.com/watch?v=Krjd_vU4Uvg

SAT Connection

Heart of Algebra

9. Understand connections between algebraic and graphical representations.

Example:

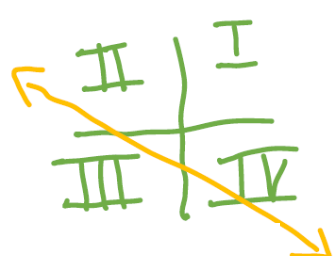
Line ℓ in the xy-plane contains points from each of Quadrants II, III, and IV, but no points from Quadrant I. Which of the following must be true?

A) The slope of line ℓ is undefined.

B) The slope of line ℓ is zero.

C) The slope of line ℓ is positive.

D) The slope of line ℓ is negative.



line ℓ is decreasing, ∴ slope is negative

Solution

Leading Term Test for Polynomial End Behavior

For any polynomial function $f(x) = a_n x^n + \cdots + a_1 x + a_0$,

$$\lim_{x\to-\infty} f(x) \quad \text{and} \quad \lim_{x\to\infty} f(x)$$

are determined by the degree n of the polynomial and its leading coefficient a_n.

highest exponent

— coefficient of term w/ highest exponent

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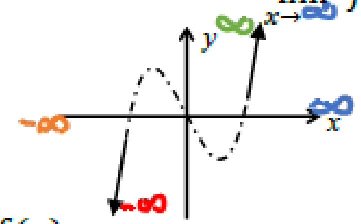
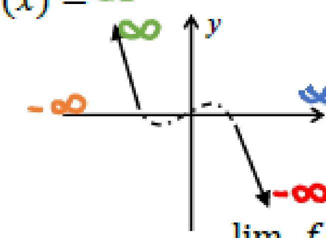
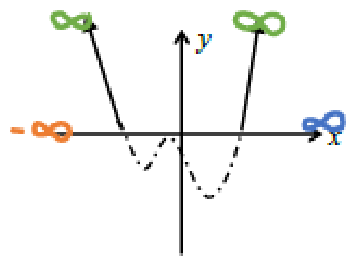
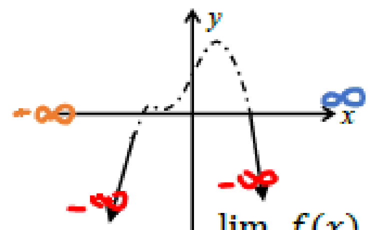
		Leading Coefficients	
		$a_n > 0$	$a_n < 0$
Degree	n is odd	 $\lim_{x\to\infty} f(x) = \infty$; $\lim_{x\to-\infty} f(x) = -\infty$	 $\lim_{x\to-\infty} f(x) = \infty$; $\lim_{x\to\infty} f(x) = -\infty$
	n is even	 $\lim_{x\to-\infty} f(x) = \infty$; $\lim_{x\to\infty} f(x) = \infty$	 $\lim_{x\to-\infty} f(x) = -\infty$; $\lim_{x\to\infty} f(x) = -\infty$

Conclusions about Leading Term Test

1. When n (degree) is even, the end behaviors are equal (the same)

2. When n is odd, the end behaviors are opposite

3. Whenever the leading coefficient is positive, $\lim_{x\to\infty} f(x) =$ ∞
   - In other words, the graph ends by approaching the positive direction.

4. Whenever the leading coefficient is negative, $\lim_{x\to\infty} f(x) =$ $-\infty$
   - In other words, the graph ends by approaching the negative direction.

Examples

Describe the end behavior of each function WITHOUT using a graphing calculator

1. $f(x) = x^4 - 2x$

L.c. positive; degree even

$\lim_{x \to -\infty} f(x) = \infty$

$\lim_{x \to \infty} f(x) = \infty$

2. $g(x) = -4x^5$

L.c. negative; degree odd

$\lim_{x \to -\infty} g(x) = \infty$

$\lim_{x \to \infty} g(x) = -\infty$

3. $h(x) = 7 - 3x^6$

L.c. negative; degree even

$\lim_{x \to -\infty} h(x) = -\infty$

$\lim_{x \to \infty} h(x) = -\infty$

4. $k(x) = -\frac{1}{2}x^2 + 5x^7$

L.c. positive; degree odd

$\lim_{x \to -\infty} k(x) = -\infty$

$\lim_{x \to \infty} k(x) = \infty$

More Practice

Leading Term Test

http://hotmath.com/hotmath_help/topics/leading-coefficient-test.html
https://www.boundless.com/algebra/textbooks/boundless-algebra-textbook/polynomials-and-rational-functions-7/graphing-polynomial-functions-346/the-leading-term-test-143-725/
https://www.khanacademy.org/math/algebra2/polynomial-functions/polynomial-end-behavior/v/polynomial-end-behavior
http://www.math.brown.edu/UTRA/polynomials.html#graphing
https://www.youtube.com/watch?v=W1mSBnu61MI
https://www.youtube.com/watch?v=WU4sufdUHqY

Homework Assignment

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SAT Connection

Solution

Choice D is correct. The quadrants of the xy-plane are defined as follows: Quadrant I is above the x-axis and to the right of the y-axis; Quadrant II is above the x-axis and to the left of the y-axis; Quadrant III is below the x-axis and to the left of the y-axis; and Quadrant IV is below the x-axis and to the right of the y-axis. It is possible for line ℓ to pass through Quadrants II, III, and IV, but not Quadrant I, only if line ℓ has negative x- and y-intercepts. This implies that line ℓ has a negative slope, since between the negative x-intercept and the negative y-intercept the value of x increases (from negative to zero) and the value of y decreases (from zero to negative); so the quotient of the change in y over the change in x, that is, the slope of line ℓ, must be negative.

Choice A is incorrect because a line with an undefined slope is a vertical line, and if a vertical line passes through Quadrant IV, it must pass through Quadrant I as well. Choice B is incorrect because a line with a slope of zero is a horizontal line and, if a horizontal line passes through Quadrant II, it must pass through Quadrant I as well. Choice C is incorrect because if a line with a positive slope passes through Quadrant IV, it must pass through Quadrant I as well.