

2.3 Polynomial Functions of Higher Degree w/Modeling

Target 2A: Graph, Solve and Analyze Polynomial Functions

Multiplicity of Zeros of Polynomials: Discovery Activity

1. Graph  $f(x) = x(x - 2)(x + 2)(x - 1)$  on a graphing calculator

- a. What are the zeros of the function?  $0, 2, -2, 1$
- b. For what value(s) of  $x$  does the graph of the function cross the  $x$ -axis?  
 $0, 2, -2, 1$
- c. For what value(s) of  $x$  does the graph of the function touch but not cross the  $x$ -axis?
- d. What degree is the polynomial?  $4$  none

2. Graph each function in the table. For each function, answer the questions asked in Question 1. Use the table below to record your results.  $2 + 1 + 1 =$

#	Function	Zeros	Cross	Touch	Degree
1	$f(x) = (x + 1)^2(x - 2)^1(x - 1)^1$	$-1, 1, 2$	$1, 2$	$-1$	$4$
2	$f(x) = (x - 2)^2(x + 1)^1(x - 1)^1$	$-1, 1, 2$	$-1, 1$	$2$	$4$
3	$f(x) = (x + 2)^2(x - 1)^2$	$-2, 1$	none	$-2, 1$	$4$
4	$f(x) = (x + 1)^3(x - 1)^1(x - 2)^1$	$-1, 1, 2$	$-1, 1, 2$	none	$5$
5	$f(x) = (x - 2)^2(x - 1)^1(x + 1)^2$	$-1, 1, 2$	$1$	$-1, 2$	$5$

3. How are the zeros of a polynomial function related to the factors of a polynomial function?

Set factors equal to zero to get zeros of a polynomial

4. How do the exponents in each term in the factored form of the polynomial function affect its graph?

(Notice: When does the graph cross the  $x$ -axis and when does the graph touch the  $x$ -axis?)

When exponent of factor is odd, function crosses  $x$ -axis.

When exponent of factor is even, function touches  $x$ -axis.

5. When a polynomial has a repeated linear factor, it has a multiple zero.

a. Write the factored form of a polynomial function that crosses the  $x$ -axis at  $x = -2$  and  $x = 5$  and touches the  $x$ -axis at  $x = 3$ .

even exponent

$$f(x) = (x - 3)^2(x + 2)(x - 5)$$

b. Which of the zeros of the function must have a multiplicity greater than 1? Explain your reasoning.

$x = 3$  must have even exponent, so  $x = 3$  must have multiplicity of at least 2. (2 is greater than 1)

6. Write two additional polynomial functions that meet the same conditions as described in Question 5. Explain what is different from your function in Question 5, and how you determined your polynomial functions.

$$g(x) = (x-3)^2(x+2)^3(x-5)$$

$$h(x) = (x-3)^4(x+2)(x-5)$$

$x-3$  must have even exponent,  
 $x+2$  and  $x-5$  must have odd exponents.

Summary of Multiplicity of Zeros of Polynomial Functions

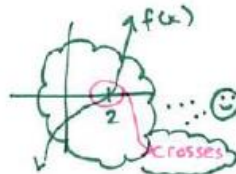
↳ repeated zero  
 $f(x) = (x-c)^m$   
 $x=c$  is a zero w/ multiplicity  $m$

$x$ -int of  $y = f(x)$   
 solutions to  $f(x) = 0$   
 roots of  $f(x) = 0$

**Odd** multiplicity → the graph crosses  $x$ -axis @

$$x = c$$

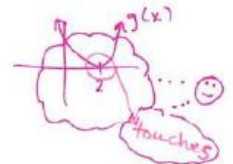
ex:  $f(x) = (x-2)^3$  ← odd



**Even** multiplicity → the graph touches  $x$ -axis @

$$x = c$$

ex:  $f(x) = (x-2)^4$  ← even



State the degree and list the zeros of the polynomial function. State the multiplicity of each zero and whether the graph crosses or touches the  $x$ -axis at the corresponding  $x$ -intercept. Then sketch the graph of the polynomial function by hand.

Example 1:

$$f(x) = x^1(x+4)^2(x-6)^1$$

degree: 4 (1+2+1) ... ☺

Zeros:  $x=0$  mult. 1, crosses  $x$ -axis  
 $x=-4$  mult. 2, touches  $x$ -axis  
 $x=6$  mult. 1, crosses  $x$ -axis



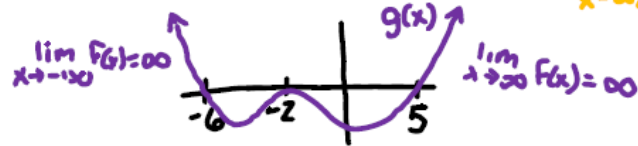
i.c. positive  
 degree even  
 end behavior =  
 $\lim_{x \rightarrow -\infty} f(x) = \infty$   
 $\lim_{x \rightarrow \infty} f(x) = \infty$

Example 2:

$$g(x) = (x-5)^3(x+6)^1(x+2)^2$$

degree: 6 (3+1+2) ... ☺

Zeros:  $x=5$  mult. 3, crosses  $x$ -axis  
 $x=-6$  mult. 1, crosses  $x$ -axis  
 $x=-2$  mult. 2, touches  $x$ -axis



i.c. positive  
 degree even,  
 end behavior =  
 $\lim_{x \rightarrow -\infty} f(x) = \infty$   
 $\lim_{x \rightarrow \infty} f(x) = \infty$

Example 3:

Write a polynomial function that has a zero with multiplicity of 2 at  $x = 3$  and a zero with multiplicity of 1 at  $x = -5$ . Then sketch the graph of the polynomial by hand.

$$f(x) = (x-3)^2(x+5)$$

odd degree  
 end behavior opposite



$$x-3=0$$

**More Practice**

**Multiplicities of Zeroes in Polynomials**

[http://www.onemathematicalcat.org/Math/Precalculus\\_obj/multZeroes.htm](http://www.onemathematicalcat.org/Math/Precalculus_obj/multZeroes.htm)

[http://stem.utm.edu/cats/index.php/Zeros and multiplicities](http://stem.utm.edu/cats/index.php/Zeros_and_multiplicities)

<http://www.coolmath.com/algebra/22-graphing-polynomials/06-zeros-multiplicities-01>

<https://www.youtube.com/watch?v=XmqJx4ujBr0>

[https://www.youtube.com/watch?v=B9SNJXvP\\_t0](https://www.youtube.com/watch?v=B9SNJXvP_t0)

**Homework Assignment**

p.193 #29,32,37,41,51,53,65,73,74