2.4 Real Zeroes of Polynomial Functions

Target 2B: Find Real and Complex Zeroes of Polynomials by Synthetic and Long Division

Review of Prior Concepts

Write the factored form of the polynomial function that crosses the x-axis @ $x = 3$ and $x = -2$ and touches the x-axis @ $x = 1$ with a degree of 6.

even exponent

odd exponent

exponents need to add to 6

$f(x) = (x-1)^4(x-3)(x+2)$

Write the factored form of the polynomial function: $f(x) = x^3 - 4x$

$f(x) = x(x^2-4)$

$f(x) = x(x-2)(x+2)$

Perform long division to find the remainder of: $1272 \div 7$

181 ← quotient

divisor → 7) 1272 ← dividend

$$\begin{array}{r} -7 \\ \hline 57 \\ -56 \\ \hline 12 \\ 7 \\ \hline 5 \end{array}$$

5 ← remainder

Recall: $\frac{1272}{7} = 181 + \frac{5}{7}$

More Practice

Factored form of Polynomials

https://www.youtube.com/watch?v=PmBNhKRhpqE

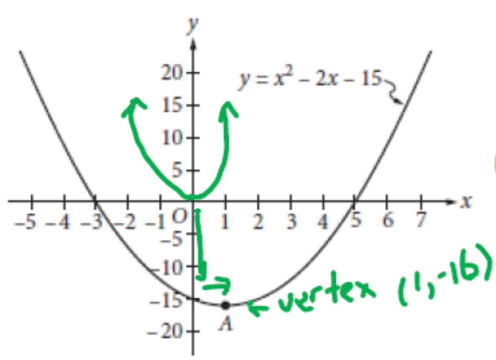
http://www.mathplanet.com/education/algebra-1/factoring-and-polynomials/polynomial-equations-in-factored-form

SAT Connection

Passport to Advanced Mathematics

11. Understand the relationship between zeros and factors of polynomials.

Example:



A) $y = (x + 3)(x - 5)$

B) $y = (x - 3)(x + 5)$

C) $y = x(x - 2) - 15$

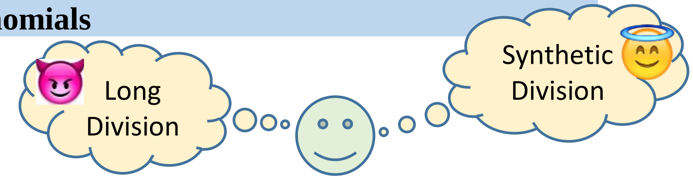
D) $y = (x - 1)^2 - 16$

Parent function x^2 shifted right 1 unit, down 16 units

Which of the following is an equivalent form of the equation of the graph shown in the xy-plane above, from which the coordinates of vertex A can be identified as constants in the equation?

Solution

Dividing Polynomials



Divide $f(x) = x^3 + 4x^2 + 7x - 9$ by $d(x) = x + 3$

<u>Long Division</u>

$$
\begin{array}{r l}
 & x^2 + x + 4 \\
x+3 \overline{)} & x^3 + 4x^2 + 7x - 9 \\
 & -(x^3 + 3x^2) \\
 & \quad x^2 + 7x \\
 & \quad -(x^2 + 3x) \\
 & \qquad 4x - 9 \\
 & \qquad -(4x + 12) \\
 & \qquad\quad -21 \leftarrow \text{remainder}
\end{array}
$$

so, $\dfrac{x^3+4x^2+7x-9}{x+3} = \boxed{x^2 + x + 4 + \dfrac{-21}{x+3}}$

<u>Synthetic Division</u>

- Set divisor equal to zero & solve for x.
- Write coefficients of dividend
  (use 0 for any missing term)

$x + 3 = 0$
$x = -3$

$$
\begin{array}{r|rrrr}
-3 & 1 & 4 & 7 & -9 \\
 & & -3 & -3 & -12 \\
\hline
 & \underbrace{1 \quad 1 \quad 4}_{\text{quotient}} & & & \underbrace{-21}_{\text{remainder}}
\end{array}
$$

multiply

multiply

multiply

① bring down L.C.

② multiply by $x = k$

③ add

④ repeat ② + ③

$\dfrac{x^3+4x^2+7x-9}{x+3} = \boxed{x^2 + x + 4 - \dfrac{21}{x+3}}$

Example: Divide $f(x) = x^3 + 3x - 4$ by $g(x) = x - 2$

$x - 2 = 0$
$x = 2$

no x^2 term

$$
\begin{array}{r|rrrr}
2 & 1 & 0 & 3 & -4 \\
 & & 2 & 4 & 14 \\
\hline
 & 1 & 2 & 7 & 10 \rightarrow \text{remainder}
\end{array}
$$

$\dfrac{x^3+3x-4}{x-2} = \boxed{x^2 + 2x + 7 + \dfrac{10}{x-2}}$

Remainder Theorem

If a polynomial $f(x)$ is divided by $x - k$, then $f(k)$ = remainder.

Example:
Find the remainder when $f(x) = x^3 + 3x - 4$ is divided by $g(x) = x - 2$

$x - 2 = 0$
$x = 2$

$f(2) = 2^3 + 3(2) - 4$
$= 8 + 6 - 4$
$= 14 - 4$
$f(2) = \underline{10}$ → remainder

To determine if $x - k$ is a factor of a polynomial $f(x)$, use either:

① Synthetic Division
② Remainder Theorem

} if remainder is ZERO, then $x - k$ is a factor

Example:
Does $f(x) = x^3 - 5x^2 - 18x + 72$ have the factor $g(x) = x - 3$?

<u>Remainder Theorem</u>

$x - 3 = 0$
$x = 3$

$f(3) = 3^3 - 5(3)^2 - 18(3) + 72$
$= 27 - 45 - 54 + 72$
$f(3) = 0$

→ since remainder is zero, then $g(x)$ is a factor of $f(x)$

<u>Synthetic Division</u>

$x - 3 = 0$
$x = 3$

$$\begin{array}{r|rrrr} 3 & 1 & -5 & -18 & 72 \\ & & 3 & -6 & -72 \\ \hline & 1 & -2 & -24 & 0 \end{array}$$

↑ remainder is zero, ∴, $g(x)$ is a factor of $f(x)$

Example:
Find two factors of $f(x) = 3x^4 - 2x^3 - 9x^2 + 4$ using the graphing calculator and find the other factors using synthetic division.

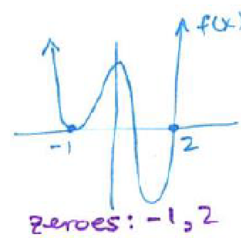
watch out: missing x-term

$$\begin{array}{r|rrrrr} -1 & 3 & -2 & -9 & 0 & 4 \\ & & -3 & 5 & 4 & -4 \\ \hline 2 & 3 & -5 & -4 & 4 & 0 \\ & & 6 & 2 & -4 & \\ \hline & 3 & 1 & -2 & 0 & \end{array}$$

→ remainder zero :)
→ remainder zero :)

$\underbrace{3x^2 + x - 2}$
$(3x-2)(x+1)$

$\boxed{\therefore,\ f(x) = (x+1)^2(x-2)(3x-2)}$



zeroes: $-1, 2$

More Practice

Real Zeros of Polynomials
http://www.coolmath.com/algebra/22-graphing-polynomials/05-real-zeros-03
https://people.richland.edu/james/lecture/m116/polynomials/zeros.html

Synthetic Division
http://mathbitsnotebook.com/Algebra2/Polynomials/POPolySynDivide.html
http://www.wtamu.edu/academic/anns/mps/math/mathlab/col_algebra/col_alg_tut37_syndiv.htm
http://www.mesacc.edu/~scotz47781/mat120/notes/divide_poly/synthetic/synthetic_division.html
https://www.youtube.com/watch?v=1byR9UEQJN0

Remainder Theorem & Factor Theorem
https://www.mathsisfun.com/algebra/polynomials-remainder-factor.html
http://mathbitsnotebook.com/Algebra2/Polynomials/PORemainderTh.html
https://www.youtube.com/watch?v=_IPqCaspZOs

Homework Assignment

p.205 #1,7,9,10,15,17,22,25,26,27

SAT Connection

Solution

Choice D is correct. Any quadratic function q can be written in the form $q(x) = a(x - h)^2 + k$, where a, h, and k are constants and (h, k) is the vertex of the parabola when q is graphed in the coordinate plane. (Depending on the sign of a, the constant k must be the minimum or maximum value of q, and h is the value of x for which $a(x - h)^2 = 0$ and $q(x)$ has value k.) This form can be reached by completing the square in the expression that defines q. The given equation is $y = x^2 - 2x - 15$, and since the coefficient of x is -2, the equation can be written in terms of $(x - 1)^2 = x^2 - 2x + 1$ as follows: $y = x^2 - 2x - 15 = (x^2 - 2x + 1) - 16 = (x - 1)^2 - 16$. From this form of the equation, the coefficients of the vertex can be read as $(1, -16)$

Choices A and C are incorrect because the coordinates of the vertex A do not appear as constants in these equations. Choice B is incorrect because it is not equivalent to the given equation.