

**2.4 Real Zeroes of Polynomial Functions**

Target 2B: Find Real and Complex Zeroes of Polynomials by Synthetic and Long Division

**SAT Connection****Passport to Advanced Mathematics****11.** Understand the relationship between zeros and factors of polynomials.

Example: For a polynomial  $p(x)$ , the value of  $p(3)$  is  $-2$ .  
Which of the following must be true about  $p(x)$  ?

- A)  $x - 5$  is a factor of  $p(x)$ .
- B)  $x - 2$  is a factor of  $p(x)$ .
- C)  $x + 2$  is a factor of  $p(x)$ .
- D) The remainder when  $p(x)$  is divided by  $x - 3$  is  $-2$ .

Solution**Rational Zeroes Theorem****Watch a video or view a website to learn about Rational Zeroes Theorem**[http://www.wtamu.edu/academic/anns/mps/math/mathlab/col\\_algebra/col\\_alg\\_tut38\\_zero1.htm](http://www.wtamu.edu/academic/anns/mps/math/mathlab/col_algebra/col_alg_tut38_zero1.htm)<https://www.youtube.com/watch?v=7p2yeuAXSCs>

Given a polynomial with integer coefficients,

$$f(x) = \underbrace{a_n}_{\text{integer}} x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + \underbrace{a_0}_{\text{integer}},$$

and  $x = \frac{p}{q}$  (in lowest terms) is a rational zero of  $f(x)$ then,  $\frac{p}{q} =$  \_\_\_\_\_

(write an example from the website/video)

*Example 1:*

*Example 2:*

Find the rational zeroes of  $f(x) = 2x^3 - 3x^2 - 23x + 12$

- Factors of the constant  $\rightarrow$
- Factors of the *l.c.*  $\rightarrow$
- Possible rational zeroes:

*Example 3:* Find the zeroes of  $f(x) = x^3 - 6x^2 + 7x + 4$  and identify as rational or irrational.

### More Practice

#### Rational Zeroes Theorem

<http://www.sparknotes.com/math/algebra2/polynomials/section4.rhtml>

<http://www.virtualnerd.com/algebra-2/polynomials/roots-zeros/rational-zero-theorem/rational-zeros-example>

[http://www.math-prof.com/Alg2/Alg2\\_Ch\\_16.asp](http://www.math-prof.com/Alg2/Alg2_Ch_16.asp)

<https://www.youtube.com/watch?v=YMyv9-9VXw4>

<https://www.youtube.com/watch?v=7mNBBBspqUc>

### Homework Assignment

p.206 #33,34,49,51,54,71,72

## SAT Connection

## Solution

**Choice D is correct.** If the polynomial  $p(x)$  is divided by  $x - 3$ , the result can be written as  $\frac{p(x)}{x - 3} = q(x) + \frac{r}{x - 3}$ , where  $q(x)$  is a polynomial and  $r$  is the remainder. Since  $x - 3$  is a degree 1 polynomial, the remainder is a real number. Hence,  $p(x)$  can be written as  $p(x) = (x - 3)q(x) + r$ , where  $r$  is a real number. It is given that  $p(3) = -2$  so it must be true that  $-2 = p(3) = (3 - 3)q(3) + r = (0)q(3) + r = r$ . Therefore, the remainder when  $p(x)$  is divided by  $x - 3$  is  $-2$ .

Choice A is incorrect because  $p(3) = -2$  does not imply that  $p(5) = 0$ . Choices B and C are incorrect because the remainder  $-2$  or its negative,  $2$ , need not be a root of  $p(x)$ .