## **Unit 2 (Chapter 2): Polynomial & Rational Functions**

# 2.4 Real Zeroes of Polynomial Functions

Target 2B: Find Real and Complex Zeroes of Polynomials by Synthetic and Long Division



#### **SAT Connection**

## **Passport to Advanced Mathematics**

**11.** Understand the relationship between zeros and factors of polynomials.

- A) x-5 is a factor of p(x).
- B) x-2 is a factor of p(x).
- C) x + 2 is a factor of p(x).

D) The remainder when p(x) is divided by x - 3 is -2.

Example: For a polynomial p(x), the value of p(3) is -2.

Which of the following must be true about p(x)?

Remainder theorem states: If a polynomial f(x) is divided by x-K, then f(K)=remainder"

K=3 remainder

Solution

#### **Rational Zeroes Theorem**



## Watch a video or view a website to learn about Rational Zeroes Theorem

http://www.wtamu.edu/academic/anns/mps/math/mathlab/col\_algebra/col\_alg\_tut38\_zero1.htm https://www.youtube.com/watch?v=7p2yeuAXSCs

Given a polynomial with integer coefficients,

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0,$$
factors of
L.c. are called

Constant are called

then  $x = \frac{p}{q}$  is a rational zero of f(x).

where 
$$\frac{p}{q} = \frac{\text{factors of the constant}}{\text{factors of the leading coefficient}}$$

(write an example from the website/video)

Example 1:

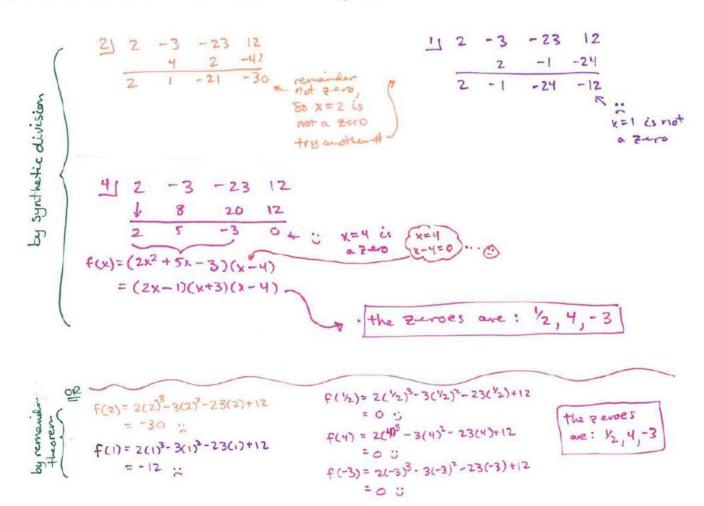
examples vary by student

## Example 2:

Find the rational zeroes of  $f(x) = 2x^3 - 3x^2 - 23x + 12$ 

- Factors of the constant → p={±1, ±2, ±3, ±4, ±6, ±12}
- Factors of the l.c. → 9 = {±1, ±2}
- Possible rational zeroes:  $\frac{P}{2} = \frac{5}{2} \pm 1, \pm \frac{1}{2}, \pm 2, \pm 3, \pm \frac{3}{2}, \pm 4, \pm 6, \pm 12$

\* choose a possible rational zero + do synthetic division or remainder theorem \*



Example 3: Find the zeroes of  $f(x) = x^3 - 6x^2 + 7x + 4$  and identify as rational or irrational.

# **More Practice**

## **Rational Zeroes Theorem**

http://www.sparknotes.com/math/algebra2/polynomials/section4.rhtml

 $\underline{http://www.virtualnerd.com/algebra-2/polynomials/roots-zeros/rational-zero-theorem/rational-zeros-\underline{example}$ 

http://www.math-prof.com/Alg2/Alg2\_Ch\_16.asp

https://www.youtube.com/watch?v=YMyv9-9VXw4

https://www.youtube.com/watch?v=7mNBBBspqUc

# **Homework Assignment**

p.206 #33,34,49,51,54,71,72

#### **SAT Connection**

#### Solution

**Choice D is correct.** If the polynomial p(x) is divided by x-3, the result can be written as  $\frac{p(x)}{x-3}=q(x)+\frac{r}{x-3}$ , where q(x) is a polynomial and r is the remainder. Since x-3 is a degree 1 polynomial, the remainder is a real number. Hence, p(x) can be written as p(x)=(x-3)q(x)+r, where r is a real number. It is given that p(3)=-2 so it must be true that -2=p(3)=(3-3)q(3)+r=(0)q(3)+r=r. Therefore, the remainder when p(x) is divided by x-3 is -2.

Choice A is incorrect because p(3) = -2 does <u>not</u> imply that p(5) = 0. Choices B and C are incorrect because the remainder -2 or its negative, 2, need not be a root of p(x).