2.5 Complex Numbers & the Fundamental Theorem of Algebra

Target 2C: Construct Polynomials given Real and/or Imaginary Zeroes
Target 2D: Understand the Fundamental Theorem of Algebra

Review of Prior Concepts

Solve each quadratic equation:

a) $x^2 = -4$

$\sqrt{x^2}=\sqrt{-4}$

$x=\pm\sqrt{-4}$

$\boxed{x=\pm 2i}$

b) $x^2 + 5 = 4x$

$x^2-4x+5=0$

$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(5)}}{2(1)}$

$=\frac{4\pm\sqrt{16-20}}{2}$

$=\frac{4\pm\sqrt{-4}}{2}$

$=\frac{4\pm 2i}{2} \rightarrow \frac{4}{2}\pm\frac{2i}{2}=\boxed{2\pm i}$

More Practice

Complex Solutions

http://www.regentsprep.org/regents/math/algtrig/ate3/quadcomlesson.htm
http://www.coolmath.com/algebra/10-complex-numbers/03-quadratic-formula-01
https://www.mathsisfun.com/numbers/complex-numbers.html

SAT Connection

Passport to Advanced Math

4. Create an equivalent form of an algebraic expression

Example: Which of the following complex numbers is equivalent to $\frac{3-5i}{8+2i}$? (Note: $i=\sqrt{-1}$)

A) $\frac{3}{8}-\frac{5i}{2}$

B) $\frac{3}{8}+\frac{5i}{2}$

C) $\frac{7}{34}-\frac{23i}{34}$

D) $\frac{7}{34}+\frac{23i}{34}$

$\frac{3-5i}{8+2i}\cdot\frac{8-2i}{8-2i}=\frac{24-6i-40i+10i^2}{64-4i^2}$

$=\frac{24-46i-10}{64+4}$

$=\frac{14-46i}{68}$

$=\frac{14}{68}-\frac{46i}{68}$

$=\frac{7}{34}-\frac{23}{34}i$

Solution

Fundamental Theorem of Algebra

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Vocabulary Term	In my own words…	Example(s)
Fundamental Theorem of Algebra	A polynomial function, f(x) with degree > 0 has # of complex zeroes = degree (some zeros can repeat)	$f(x) = (x-2)(x+3)$ degree: 2 # of zeroes: 2 $g(x) = (x-2)^2(x+3)$ degree: 3 # of zeroes: 3 (one zero repeats)

Examples:

Using your graphing calculator, find the complex zeroes and write the polynomial in factored form:

1) $f(x) = 4x + 3$

degree: 1

zero: −0.75

$f(x) = 4(x + .75)$

2) $g(x) = x^2 - 4$

degree: 2

zeroes: −2, 2

$g(x) = (x+2)(x-2)$

3) $h(x) = 2x^3 + 3x^2 - 11x - 6$

degree: 3

zeroes: −3, −0.5, 2

$x = -3$, $x + 3 = 0$

$x = -.5$, $x = -\frac{1}{2}$, $x + \frac{1}{2} = 0$, $2x + 1 = 0$

$x = 2$, $x - 2 = 0$

$h(x) = (x+3)(2x+1)(x-2)$

4) $f(x) = x^3 - 2x^2 + x - 2$

degree: 3

zeroes: 2, i, −i

synthetic division w/ zero from graph calc

$$\begin{array}{r|rrrr} 2 & 1 & -2 & 1 & -2 \\ & \downarrow & 2 & 0 & 2 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

☺

$\underbrace{1 \quad 0 \quad 1}$

$x^2 + 1 = 0$

$\sqrt{x^2} = \sqrt{-1}$

$x = \pm i$

Complex Conjugate Zeros

If $a + bi$ is a zero of $f(x)$, then $a - bi$ is also a zero of $f(x)$

****Imaginary Zeroes are always conjugate pairs *****

Examples:

Write a polynomial in standard from with the following zeroes:

1) $4, 2i$

↳ also, $-2i$ (conjugate pair)

$x = 4$ $\quad x = 2i$ $\quad x = -2i$

$x - 4 = 0$ $\quad x - 2i = 0$ $\quad x + 2i = 0$

$$f(x) = (x-4)(x-2i)(x+2i)$$

→ multiply conjugate pairs 1st

$$= (x-4)(x^2 + 2ix - 2ix - 4i^2)$$
$$= (x-4)(x^2 - 4(-1))$$
$$= (x-4)(x^2 + 4)$$
$$\boxed{f(x) = x^3 - 4x^2 + 4x - 16}$$

2) 3 (multiplicity 2), $1 - i$ (multiplicity 1)

→ also, $1 + i$ (conjugate pair)

$x = 3$ $\quad x = 1 - i$ $\quad x = 1 + i$

$x - 3 = 0$ $\quad x - 1 + i = 0$ $\quad x - 1 - i = 0$

$$f(x) = (x-3)^2(x-1+i)(x-1-i)$$
$$= (x-3)^2(x^2 - x - ix - x + 1 + i + ix - i - i^2)$$
$$= (x-3)^2(x^2 - 2x + 1 - (-1))$$
$$= (x-3)^2(x^2 - 2x + 2)$$

$(x-3)(x-3)$
$x^2 - 3x - 3x + 9$

$$= (x^2 - 6x + 9)(x^2 - 2x + 2)$$
$$= x^4 - 2x^3 + 2x^2 - 6x^3 + 12x^2 - 12x + 9x^2 - 18x + 18$$
$$\boxed{f(x) = x^4 - 8x^3 + 23x^2 - 30x + 18}$$

7) Given $f(x) = x^4 - 2x^3 + 5x^2 + 10x - 50$ has a zero of $1 + 3i$. Find all of the zeroes and write a linear factorization of $f(x)$.

↳ $1 - 3i$ is another zero

synthetic division

$1+3i$	1	-2	5	10	-50
	↓	$1+3i$	-10	$-5-15i$	50
$1-3i$	1	$-1+3i$	-5	$5-15i$	0
	↓	$1-3i$	0	$-5+15i$	
	1	0	-5	0	

$(1+3i)(-1+3i)$
$-1 + 3i - 3i + 9i^2$
$-1 - 9 = -10$

$(1+3i)(5-15i)$
$5 - 15i + 15i - 45i^2$
$5 + 45 = 50$

$$x^2 - 5 = 0$$
$$x^2 = 5$$
$$x = \pm\sqrt{5}$$

$x = 1 + 3i$ $\quad x = 1 - 3i$ $\quad x = \sqrt{5}$ $\quad x = -\sqrt{5}$

$x - 1 - 3i = 0$ $\quad x - 1 + 3i = 0$ $\quad x - \sqrt{5} = 0$ $\quad x + \sqrt{5} = 0$

$$\boxed{f(x) = (x - 1 - 3i)(x - 1 + 3i)(x - \sqrt{5})(x + \sqrt{5})}$$

More Practice

Fundamental Theorem of Algebra

https://www.khanacademy.org/math/algebra2/polynomial-functions/fundamental-theorem-of-algebra/v/fundamental-theorem-of-algebra-intro
https://www.mathsisfun.com/algebra/fundamental-theorem-algebra.html
https://www.youtube.com/watch?v=NZS3T43NBvE
https://www.youtube.com/watch?v=PQr0yVq5ysc
https://www.youtube.com/watch?v=gyksK76Dg1c

Homework Assignment

p.215 #3,5,9,11,15,17,20,27,31

SAT Connection
Solution

Choice C is correct. To perform the division $\frac{3-5i}{8+2i}$, multiply the numerator and denominator of $\frac{3-5i}{8+2i}$ by the conjugate of the denominator, $8-2i$. This gives $\frac{(3-5i)(8-2i)}{(8+2i)(8-2i)} = \frac{24-6i-40i+(-5i)(-2i)}{8^2-(2i)^2}$. Since $i^2=-1$, this can be simplified to $\frac{24-6i-40i-10}{64+4} = \frac{14-46i}{68}$, which then simplifies to $\frac{7}{34} - \frac{23i}{34}$.

Choices A and B are incorrect and may result from misconceptions about fractions. For example, $\frac{a+b}{c+d}$ is equal to $\frac{a}{c+d} + \frac{b}{c+d}$, not $\frac{a}{c} + \frac{b}{d}$. Choice D is incorrect and may result from a calculation error.