DATE: ___________

2.5 Complex Zeros and Fundamental Theorem of Algebra (More Practice)

(Target 2C/2D)

1. Write a polynomial in standard form with the following zeroes:

a) $3+i, -2$ conjugate pair

$x = 3+i$ $\quad x = 3-i$ $\quad x = -2$

$x-3-i=0$ $\quad x-3+i=0$ $\quad x+2=0$

$f(x) = (x-3-i)(x-3+i)(x+2)$

$= (x^2-3x+ix-3x+9-3i-ix+3i-i^2)(x+2)$

$= (x^2-6x+9-i^2)(x+2)$

$= (x^2-6x+9-(-1))(x+2)$

$= (x^2-6x+10)(x+2)$

$= x^3+2x^2-6x^2-12x+10x+20$

$\boxed{f(x) = x^3-4x^2-2x+20}$

b) $3i$ (multiplicity 1), 1 (multiplicity 2), 0 (multiplicity 3)

$x = 3i$ $\quad x = -3i$ $\quad x = 1$ $\quad x = 0$

$x-3i=0$ $\quad x+3i=0$ $\quad x-1=0$

$f(x) = (x-3i)(x+3i)(x-1)^2 \cdot x^3$

$= (x^2-3ix+3ix-9i^2)(x-1)^2 \cdot x^3$

$= (x^2-9(-1))(x-1)^2 \cdot x^3$

$= (x^2+9)(x-1)^2 \cdot x^3$

$(x^2+9)(x-1)(x-1)x^3$

$= (x^2+9)(x^2-x-x+1)x^3$

$= (x^2+9)(x^2-2x+1)x^3$

$= (x^4+9x^2-2x^3+x^2-18x+9)(x^3)$

$= (x^4-2x^3+10x^2-18x+9)x^3$

$= \boxed{x^7-2x^6+10x^5-18x^4+9x^3}$

2. Identify the zeroes and x-intercepts of the polynomial.

a) $f(x) = (x-3)^2(x-1-i)(x-1+i)$

$x-3=0$ $\quad x-1-i=0$ $\quad x-1+i=0$

$x=3$ $\quad x=1+i$ $\quad x=1-i$

$\boxed{\text{zeroes: } 3, 1+i, 1-i}$

$\boxed{x\text{-int: } 3}$

b) $g(x) = x(x-4i)(x+4i)(x+1)^2$

$x=0$ $\quad x-4i=0$ $\quad x+4i=0$ $\quad x+1=0$

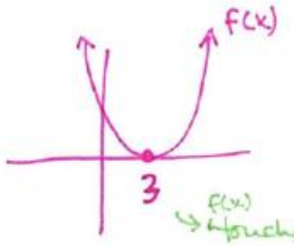
$x=4i$ $\quad x=-4i$ $\quad x=-1$

$\boxed{\text{zeroes: } 0, -1, \pm 4i}$

$\boxed{x\text{-int: } 0, -1}$

Draw a picture of (or explain why you are not able to draw) each of the following:

a) a quadratic function having only one real number root



$f(x) = (x-3)^2$

f(x) touches @ $x=3$ (multiplicity of 2)

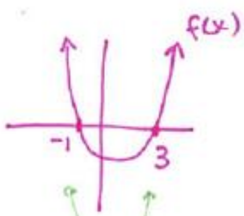
b) a quadratic function having only one complex root.

not possible

a quadratic function must have 2 roots.

If one root is complex, then the 2nd root is its conjugate pair. $\therefore$, function must have 2 complex roots.

c) a quadratic function with two real roots



$f(x) = (x+1)(x-3)$

f(x) crosses @ $x=-1$ (multiplicity 1)
$x=3$ (multiplicity 1)

d) a quadratic function with two complex roots.

$f(x) = x^2+4$

f(x) does not touch nor cross the x-axis (roots are imaginary $x = \pm 2i$)