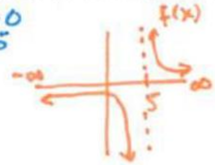
2.6 Rational Functions

Target 2E: Graph, Solve and Analyze Rational Functions

Review of Prior Concepts

Using your graphing calculator, find the domain and describe the end behavior:

a) $f(x) = \frac{1}{x-5}$ $x-5 \neq 0$, $x \neq 5$

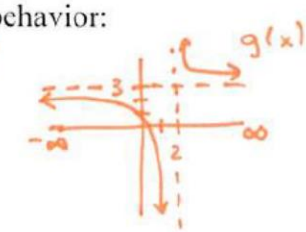


$\lim_{x \to \infty} f(x) = 0$

$\lim_{x \to -\infty} f(x) = 0$

Domain: $(-\infty, -5) \cup (-5, \infty)$

b) $g(x) = \frac{3x-5}{x-2}$ $x-2 \neq 0$, $x \neq 2$



$\lim_{x \to \infty} g(x) = 3$

$\lim_{x \to -\infty} g(x) = 3$

Domain: $(-\infty, 2) \cup (2, \infty)$

More Practice

Domain & End Behavior

http://www.coolmath.com/algebra/15-functions/06-finding-the-domain-01
https://www.khanacademy.org/math/algebra/algebra-functions/domain-and-range/v/domain-of-a-function-intro
https://youtu.be/Krjd_vU4Uvg
https://youtu.be/PQ85Y1jsVzQ

SAT Connection

Passport to Advanced Math

10. Interpret parts of nonlinear expressions in terms of their context

Example:

$$h(x) = \frac{1}{(x-5)^2 + 4(x-5) + 4}$$

For what value of x is the function h above undefined?

h is undefined when denominator = 0

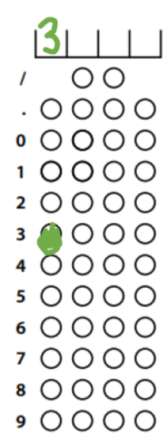
$(x-5)^2 + 4(x-5) + 4 = 0$

$x^2 - 10x + 25 + 4x - 20 + 4 = 0$
$x^2 - 6x + 9 = 0$
$(x-3)^2 = 0$
$x - 3 = 0$
$x = 3$

OR

let $y = x - 5$
$y^2 + 4y + 4 = 0$
$(y+2)(y+2) = 0$
$y = -2$

$y = x - 5$
$-2 = x - 5$
$3 = x$



NOTE: You may start your answers in any column, space permitting. Columns you don't need to use should be left blank.

Solution

Analyzing Graphs of Rational Functions

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Vocabulary Term	In my own words…	Example(s)
Rational Function	function written as a ratio (fraction) $\frac{f(x)}{g(x)}$ where $f(x)$ & $g(x)$ are polynomials	$h(x)=\frac{3x}{x-2}$ $d(x)=\frac{5x^2+3x-4}{x^2+2}$

Describe the behavior of the graphs of the rational functions at the *x*-values not in the domain.

a) $f(x)=\frac{1}{x-5}$

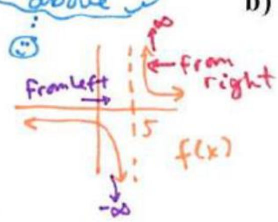
see drawing above

from left 5^-

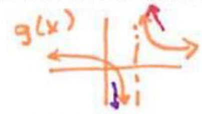
$\lim_{x\to 5^-} f(x) = -\infty$

from right 5^+

$\lim_{x\to 5^+} f(x) = \infty$



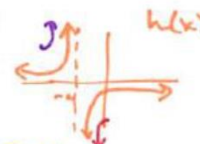
b) $g(x)=\frac{3x-5}{x-2}$



$\lim_{x\to 2^-} g(x) = -\infty$

$\lim_{x\to 2^+} g(x) = \infty$

c) $h(x)=\frac{-5}{x+4}$



$\lim_{x\to -4^-} h(x) = \infty$

$\lim_{x\to -4^+} h(x) = -\infty$

Vertical and Horizontal Asymptotes

Recall:

Vertical asymptotes occur when denominator = 0

Horizontal asymptotes are found from end behavior

Using your graphing calculator, find the vertical and horizontal asymptotes.

a) $f(x)=\frac{3x-5}{x-4}$

V.A. @ $x=4$

H.A. @ $y=3$

b) $g(x)=\frac{3x^2-5}{x^2-4}$

V.A. @ $x=2$, $x=-2$

H.A. @ $y=3$

Without using your graphing calculator, find the vertical and horizontal asymptotes algebraically.

a) $f(x)=\frac{3x-5}{x-4}$

V.A. $x-4=0$

$x=4$

H.A. @ $y=3$

$(x-4)\overline{)3x-5}$

$$\begin{array}{r|rr} 4 & 3 & -5 \\ & \downarrow & 12 \\ \hline & 3 & 7 \end{array}$$

← remainder

horizontal asymptote

$f(x)=3+\frac{7}{x-4}$

b) $g(x)=\frac{3x^2-5}{x^2-4}$

V.A. $x^2-4=0$

$(x-2)(x+2)=0$

$x=2$, $x=-2$

H.A. $y=3$

$$\begin{array}{r} 3 \\ x^2-4\overline{)3x^2-5} \\ -(3x^2-12) \\ \hline 7 \end{array}$$

← remainder

horizontal asymptote

$g(x)=3+\frac{7}{x^2-4}$

Using your graphing calculator, find the horizontal asymptotes (if any).

a) $f(x) = \frac{3x^2-5x+1}{x^2-4}$ **b)** $g(x) = \frac{3x-5}{x^2-4}$ **c)** $h(x) = \frac{3x^2-5x+1}{x-4}$

H.A. @ y = 3

H.A. @ y = 0

No H.A.

Can you find a pattern? If yes, then find the horizontal asymptotes (if any) without using your graphing calculator.

a) $f(x) = \frac{2x^3+x^2-5x+1}{x^3-4}$ **b)** $g(x) = \frac{2x-5}{x^3-4}$ **c)** $h(x) = \frac{2x^3+x^2-5x+1}{x-4}$

degree of num = degree of denom.
∴, H.A. @ y = 2

degree of numer < degree of denom.
∴, H.A. @ y = 0

degree of numer > degree of denom.
∴, no H.A.

Conclusion about Horizontal Asymptotes:

① If degree of numerator = degree of denominator,
H.A. @ $y = \frac{L.C. of\ polynomial\ (in\ numerator)}{L.C. of\ polynomial\ (in\ denominator)}$, **in other words y = ratio of L.C.'s**

② If degree of numerator < degree of denominator,
H.A. @ y = 0

③ If degree of numerator > degree of denominator,
no H.A.

More Practice

Vertical Asymptotes

http://www.sosmath.com/calculus/limcon/limcon04/limcon04.html
https://www.khanacademy.org/math/algebra2/rational-expressions-equations-and-functions/discontinuities-of-rational-functions/v/analyzing-vertical-asymptotes-of-rational-functions
https://www.youtube.com/watch?v=cIkIKyRsybY
https://www.youtube.com/watch?v=ALFNl6QHbVU

Horizontal Asymptotes

http://www.coolmath.com/precalculus-review-calculus-intro/precalculus-algebra/18-rational-functions-finding-horizontal-slant-asymptotes-01
http://www.softschools.com/math/calculus/finding_horizontal_asymptotes_of_rational_functions/
https://www.youtube.com/watch?v=E0iNtii45KA

Homework Assignment

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SAT Connection

Solution

The correct answer is 3. The function $h(x)$ is undefined when the denominator of $\dfrac{1}{(x-5)^2+4(x-5)+4}$ is equal to zero. The expression $(x-5)^2 + 4(x-5)+4$ is a perfect square: $(x-5)^2+4(x-5)+4 = ((x-5)+2)^2$, which can be rewritten as $(x-3)^2$. The expression $(x-3)^2$ is equal to zero if and only if $x = 3$. Therefore, the value of x for which $h(x)$ is undefined is 3.