

2.7 Solving Equations in One Variable

Target 2E: Graph, Solve and Analyze Rational Functions

Review of Prior Concepts

Find the least common denominator (LCD) in the following equations

a) $\frac{1}{3x} + \frac{5}{9x^2} = \frac{2}{27}$

$\frac{1}{3x} + \frac{5}{3 \cdot 3x} = \frac{2}{3 \cdot 3 \cdot 3}$

$3 \cdot 3 \cdot 3 \cdot x \cdot x$

LCD: $27x^2$

b) $2 - \frac{3}{x+4} = \frac{12}{x^2+4x}$

$2 - \frac{3}{x+4} = \frac{12}{x(x+4)}$

$x(x+4)$

LCD: $x(x+4)$

c) $\frac{3x}{x+2} + \frac{2}{x-1} = \frac{5}{x^2+x-2}$

$\frac{3x}{x+2} + \frac{2}{x-1} = \frac{5}{(x+2)(x-1)}$

$(x+2)(x-1)$

LCD: $(x+2)(x-1)$

More Practice

Least Common Denominators

<https://www.mathsisfun.com/least-common-denominator.html>

<http://www.virtualnerd.com/algebra-1/rational-expressions-functions/add-subtract/add-subtract-unlike-denominators/find-least-common-denominator-example>

<https://www.youtube.com/watch?v=bDIKHlui1E>



SAT Connection

Passport to Advanced Math

9. Rewrite simple rational expressions

Example: If $x > 3$, which of the following is equivalent

to $\frac{1}{\frac{1}{x+2} + \frac{1}{x+3}}$?

A) $\frac{2x+5}{x^2+5x+6}$

B) $\frac{x^2+5x+6}{2x+5}$

C) $2x+5$

D) x^2+5x+6

$$\begin{aligned} &= \frac{1}{\frac{(x+3)}{(x+3)} \frac{1}{x+2} + \frac{1}{x+3} \frac{(x+2)}{(x+2)}} \\ &= \frac{1}{\frac{x+3}{(x+3)(x+2)} + \frac{x+2}{(x+3)(x+2)}} \\ &= \frac{1}{\frac{2x+5}{(x+3)(x+2)}} \\ &= \frac{(x+3)(x+2)}{2x+5} \\ &= \frac{x^2+5x+6}{2x+5} \end{aligned}$$

reciprocal 😊

OR

$$\begin{aligned} &= \frac{1}{\left(\frac{1}{x+2} + \frac{1}{x+3}\right)(x+3)(x+2)} \\ &= \frac{(x+3)(x+2)}{\frac{(x+3)(x+2)}{x+2} + \frac{(x+3)(x+2)}{x+3}} \\ &= \frac{(x+3)(x+2)}{x+3 + x+2} \\ &= \frac{x^2+5x+6}{2x+5} \end{aligned}$$

multiply by LCD 😊

reduce 😊

Solution

Solving Rational Equations



Example

- ① Multiply by the Least Common Denominator, LCD.
(to eliminate the fractions/rational expressions)

$$x + \frac{3}{x} = 4$$

LCD: x

$$x(x) + \frac{3}{x}(x) = 4(x)$$

$$x^2 + 3 = 4x$$

$$x^2 - 4x + 3 = 0$$

$$(x-3)(x-1) = 0$$

$$x = 3 \quad x = 1$$

- ② Solve for x .

- ③ Check for extraneous solutions.

$$\text{check: } 3 + \frac{3}{3} = 4 \quad 1 + \frac{3}{1} = 4$$

$$3 + 1 = 4 \quad 1 + 3 = 4$$

$$4 = 4 \checkmark$$

$$4 = 4 \checkmark$$

both check...
so both $x=1, x=3$
work

Examples:

a) $\frac{3}{x+1} + \frac{2}{x} = 2$ LCD: $x(x+1)$

$$x(x+1)\left(\frac{3}{x+1}\right) + \frac{2}{x}(x)(x+1) = 2(x)(x+1)$$

$$x(\cancel{x+1})\left(\frac{3}{\cancel{x+1}}\right) + \frac{2}{\cancel{x}}(\cancel{x})(x+1) = 2(x)(x+1)$$

$$3x + 2x + 2 = 2(x^2 + x)$$

$$5x + 2 = 2x^2 + 2x$$

$$0 = 2x^2 - 3x + 2$$

$$0 = (2x+1)(x-2)$$

$$x = -\frac{1}{2}, x = 2$$

$$\text{check: } -\frac{3}{-\frac{1}{2}+1} + \frac{2}{-\frac{1}{2}} = 2 \quad \frac{3}{2+1} + \frac{2}{2} = 2$$

$$\frac{3}{\frac{1}{2}} + 2 \cdot \frac{-2}{1} \quad \frac{3}{3} + 1$$

$$3 \cdot 2 - 4 \quad 1 + 1$$

$$6 - 4 \quad 2$$

b) $2 - \frac{3}{x+4} = \frac{12}{x^2+4x}$

$$x(x+4) \cdot 2 - \frac{3}{x+4} x(x+4) = \frac{12}{x(x+4)} \cdot x(x+4)$$

$$x(x+4) \cdot 2 - \frac{3}{\cancel{x+4}} \cdot x(\cancel{x+4}) = \frac{12}{\cancel{x(x+4)}} \cdot \cancel{x(x+4)}$$

$$(x^2+4x)2 - 3x = 12$$

$$2x^2+8x-3x=12$$

$$2x^2+5x=12$$

$$2x^2+5x-12=0$$

$$(2x-3)(x+4)=0$$

$$x = \frac{3}{2}, x = -4 \text{ extraneous}$$

$$\text{check: } 2 - \frac{3}{\frac{3}{2}+4} = \frac{12}{(\frac{3}{2})^2+4(\frac{3}{2})}$$

$$\frac{16}{11} = \frac{16}{11}$$

✓

$$2 - \frac{3}{-4+4} = \frac{12}{(-1)^2+4(-1)}$$

$$2 - \frac{3}{0} = \frac{12}{0}$$

undefined
doesn't check

Solve the equation algebraically. Support your answer graphically.

1. $\frac{4x}{x+4} + \frac{3}{x-1} = \frac{15}{x^2+3x-4}$

LCD: $(x+4)(x-1)$

$(x+4)(x-1) \cdot \frac{4x}{x+4} + \frac{3}{x-1} \cdot (x+4)(x-1) = \frac{15}{(x+4)(x-1)} \cdot (x+4)(x-1)$

~~$(x+4)(x-1) \cdot \frac{4x}{x+4} + \frac{3}{x-1} \cdot (x+4)(x-1) = \frac{15}{(x+4)(x-1)}$~~

$(x-1)(4x) + 3(x+4) = 15$

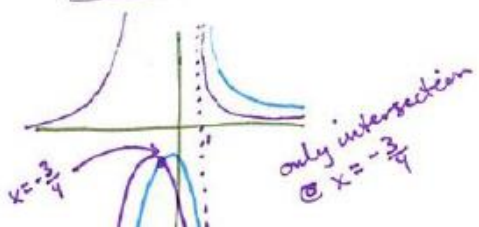
$4x^2 - 4x + 3x + 12 = 15$

$4x^2 - x + 12 = 15$

$4x^2 - x - 3 = 0$

$(4x+3)(x-1) = 0$

$x = -\frac{3}{4}, x = 1$ extraneous



2. $\frac{3}{x+2} + \frac{6}{x^2+2x} = \frac{3-x}{x}$

LCD: $x(x+2)$

$x(x+2) \cdot \frac{3}{x+2} + \frac{6}{x(x+2)} \cdot x(x+2) = \frac{3-x}{x} \cdot x(x+2)$

~~$x(x+2) \cdot \frac{3}{x+2} + \frac{6}{x(x+2)} \cdot x(x+2) = \frac{3-x}{x} \cdot x(x+2)$~~

$3x + 6 = (3-x)(x+2)$

$3x + 6 = 3x + 6 - x^2 - 2x$

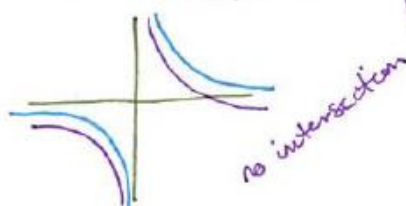
$3x + 6 = -x^2 + x + 6$

$x^2 + 2x = 0$

$x(x+2) = 0$

$x = 0, x = -2$ extraneous

\therefore no solution



Application of Rational Functions



Example: Consider all rectangles with an area of 182 square feet. Let x be the length of one side of such a rectangle.

a) Express the perimeter P as a function of x .

$x \rightarrow$ length

$A = lw$

$P = 2l + 2w$

$w \rightarrow$ width

$182 = x \cdot w$

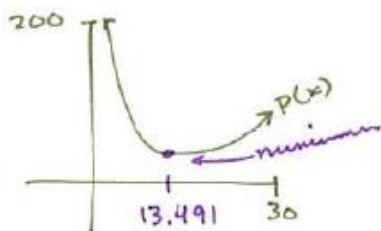
$P = 2x + 2w$

$\frac{182}{x} = w$

$P = 2x + 2\left(\frac{182}{x}\right)$

$P(x) = 2x + \frac{364}{x}$

b) Find the dimensions of the rectangle that has the least perimeter. What is the least perimeter?



$x = 13.491$

$w = \frac{182}{13.491} = 13.491$

rectangle with least perimeter : 13.491 ft x 13.491 ft

least Perimeter : $2(13.491) + 2(13.491)$

$= 53.963 \text{ ft}$

More Practice**Solving Rational Equations**

http://www.montereyinstitute.org/courses/Algebra1/COURSE_TEXT_RESOURCE/U11_L2_T1_text_final.html

<http://www.regentsprep.org/regents/math/algtrig/ate11/rationalequationsles.htm>

<http://www.mathplanet.com/education/algebra-1/rational-expressions/solving-rational-expressions>

<https://www.youtube.com/watch?v=zx82WVQrOCE>

<https://www.khanacademy.org/math/algebra-home/alg-rational-expr-eq-func/alg-modeling-with-rational-functions/v/applying-rational-equations-1>

http://www.mhhe.com/math/devmath/streeter/ia/graphics/streeter5ia/ch07/others/strI_7.6.pdf

<https://www.youtube.com/watch?v=-59ijFaVzpU>

<https://www.youtube.com/watch?v=UcTy7RcZLiw>

<https://www.youtube.com/watch?v=04TKMJVEsXc>

Homework Assignment

p.232 #11,13,16,29,39,40

SAT Connection**Solution**

Choice B is correct. To rewrite $\frac{1}{\frac{1}{x+2} + \frac{1}{x+3}}$, multiply by $\frac{(x+2)(x+3)}{(x+2)(x+3)}$.

This results in the expression $\frac{(x+2)(x+3)}{(x+3) + (x+2)}$, which is equivalent to the expression in choice B.

Choices A, C, and D are incorrect and could be the result of common algebraic errors that arise while manipulating a complex fraction.