3.1 Exponential & Logistic Functions

Target 3A: Identify and analyze properties of exponential, logarithmic, and logistic functions and their graphs

Review of Prior Concepts

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Which of the following functions are exponential functions? Explain why.

1) $f(x) = x^8$
not exponential b/c base is a variable exponent is a constant

2) $g(x) = 3^x$
exponential b/c base is a constant exponent is a variable

3) $h(x) = 5^x$
exponential b/c base is a constant exponent is a variable

4) $k(x) = 4^2$
not exponential b/c exponent is a constant

More Practice

Introduction to Exponential Functions

http://www.virtualnerd.com/algebra-2/exponential-logarithmic-functions/exponentials/exponential-functions/function-definition
https://www.khanacademy.org/math/algebra/introduction-to-exponential-functions/exponential-growth-and-decay/v/exponential-growth-functions
https://www.youtube.com/watch?v=jnOwrj8OvYI

SAT Connection

Passport to Advanced Math

14. Use structure to isolate or identify a quantity of interest in an expression

Example: If $3x - y = 12$, what is the value of $\frac{8^x}{2^y}$?

A) 2^{12}

B) 4^4

C) 8^2

D) The value cannot be determined from the information given.

$3x - y = 12$
$3x = 12 + y$
$3x - 12 = y$

$$\frac{8^x}{2^y} = \frac{8^x}{2^{3x-12}} = \frac{8^x}{2^{3x} \cdot 2^{-12}} = \frac{8^x}{8^x \cdot 2^{-12}} = \frac{1}{2^{-12}} = 2^{12}$$

Solution

Exponential Functions

a, *b*, and *k* are real number constants,

$f(x) = a \cdot b^x$ — initial value (when x = 0): a; base: b

$f(x) = a \cdot e^{kx}$ — initial value (when x = 0): a; base: e

Exponential Function	Exponential Growth		Exponential Decay	
	Conditions	*Example*	*Conditions*	*Example*
$f(x) = a \cdot b^x$	$a > 0$ and $b > 1$	$f(x) = 2 \cdot 3^x$ $f(x) = 5^x$	$a > 0$ and $0 < b < 1$	$f(x) = 2 \cdot (\frac{1}{3})^x$ $f(x) = (0.5)^x$
$f(x) = a \cdot e^{kx}$	$a > 0$ and $k > 0$	$f(x) = 2e^{3x}$ $f(x) = e^x$	$a > 0$ and $k < 0$	$f(x) = 2e^{-3x}$ $f(x) = e^{-x}$

Example 1:

Identify if the function is exponential.
If yes, determine if exponential growth or exponential decay and describe its end behavior.

a) $f(x) = 3^{-x}$

exponential function

$f(x) = (3^{-1})^x = (\frac{1}{3})^x$

exponential decay b/c $b = \frac{1}{3}$, $a = 1$

$\lim_{x \to \infty} f(x) = 0$

$\lim_{x \to -\infty} f(x) = \infty$

b) $g(x) = (0.5)^{-x}$

exponential function

$g(x) = (\frac{1}{2})^{-1x} = ((\frac{1}{2})^{-1})^x = 2^x$

exponential growth b/c $b = 2$, $a = 1$

$\lim_{x \to \infty} g(x) = \infty$

$\lim_{x \to -\infty} g(x) = 0$

c) $h(x) = x^{-3}$

not an exponential function

d) $f(x) = 3e^{2x}$

exponential function

exponential growth b/c $a = 3$ and $k = 2$

$\lim_{x \to \infty} f(x) = \infty$

$\lim_{x \to -\infty} f(x) = 0$

Example 2:

Determine a formula for the exponential function whose values are given.
Use the model to predict the population (in millions) for 2010.

Year	1900	1910	1920	1930	1940	1950	1960	1970	1980	1990	2000
Population (in millions)	76.2	92.2	106.0	123.2	132.2	151.3	179.3	203.3	226.5	248.7	281.4

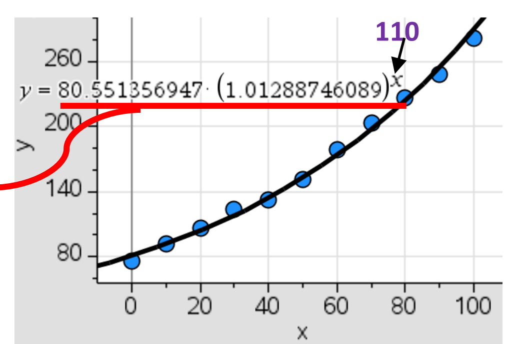
$y = 80.551(1.013)^x$

$2010 - 1900 = 110$

$y(110) = 80.551(1.013)^{110}$
$= 329.461$

or 329,461,000

store values to get more accurate answer



Logistic Growth Functions

a, *b*, *c*, and *k* are positive constants,

$$f(x) = \frac{c}{1+a\cdot b^x}$$

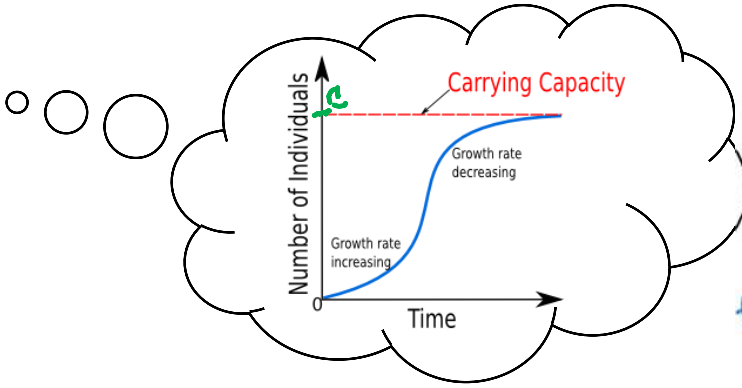
limit to growth

$0<b<1$

$$f(x) = \frac{c}{1+a\cdot e^{-kx}}$$

limit to growth

Science Connection

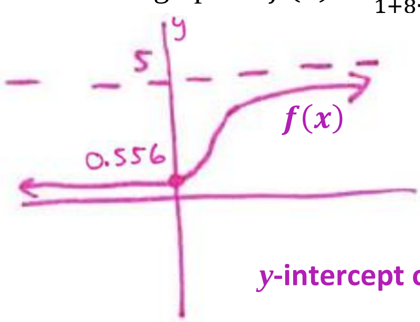


limit to growth is carrying capacity which is a H.A.

H.A. @ $y=0$ and $y=c$

Examples

1. Sketch the graph of $f(x) = \frac{5}{1+8\cdot 0.2^x}$. Identify the horizontal asymptotes and the *y*-intercept.



H.A. @ $y=0$ and $y=5$

y-intercept: 0.556

***y*-intercept occurs when *x* = 0:** $f(0) = \frac{5}{1+8\cdot 0.2^0} = \frac{5}{1+8(1)} = \frac{5}{9}$

#55

$$P(t) = \frac{12.79}{1+2.402e^{-.0309t}}$$

when → $t=?$
Pop = 10 million

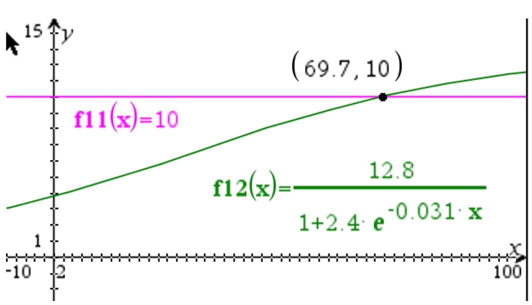
$$10 = \frac{12.79}{1+2.402e^{-.0309t}}$$

graph + get intersection...

$t = 70$ years

t is # years since 1900, *(April 1)*

So population is 10 million in $\boxed{1970}$



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Columbus (0, 632,910) 1990 — Initial population (value)

(21, 797,434) 2011

Want exponential growth:

$f(x) = a\cdot b^x$ ↳ Initial value

$f(x) = 632{,}910\cdot b^x$ Use ($\underset{x}{21}$, $\underset{f(x)}{797{,}434}$)

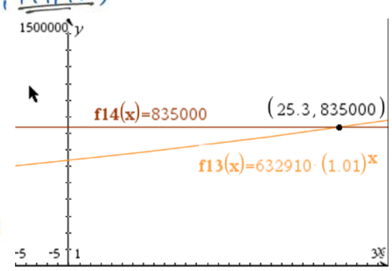
$797{,}434 = 632{,}910\cdot b^{21}$

$\frac{797{,}43}{632{,}910} = b^{21}$

$\sqrt[21]{\frac{797{,}434}{632{,}910}} = \sqrt[21]{b^{21}}$

$1.011 = b$

$\therefore$, $f(x) = 632{,}910(1.011)^x$



The problem asks us to find when pop. passes 800,000 but that's not such a good # (can you see why?). Let's see when it surpasses 835,000 instead:

$835{,}000 = 632{,}910(1.011)^x$

Graph and get intersection

$x \approx 25.3$ $\therefore$, In $1990+25 = \boxed{2015}$

More Practice

Exponential Functions

https://www.mathsisfun.com/sets/function-exponential.html
https://www.khanacademy.org/math/algebra/introduction-to-exponential-functions
http://www.regentsprep.org/regents/math/algtrig/ATP8b/exponentialfunction.htm
https://www.youtube.com/watch?v=PEtIQqvIoGU
https://www.youtube.com/watch?v=hx_h0_eo8ew

Logistic Functions

http://www.classzone.com/eservices/home/pdf/student/LA208HAD.pdf
https://www.youtube.com/watch?v=O0j4rjTM88Q

Homework Assignment

p.262 #31,33,41,43,45,46,56,57

SAT Connection
Solution

Choice A is correct. One approach is to express $\frac{8^x}{2^y}$ so that the numerator and denominator are expressed with the same base. Since 2 and 8 are both powers of 2, substituting 2^3 for 8 in the numerator of $\frac{8^x}{2^y}$ gives $\frac{(2^3)^x}{2^y}$, which can be rewritten as $\frac{2^{3x}}{2^y}$. Since the numerator and denominator of $\frac{2^{3x}}{2^y}$ have a common base, this expression can be rewritten as 2^{3x-y}. It is given that $3x - y = 12$, so one can substitute 12 for the exponent, $3x - y$, giving that the expression $\frac{8^x}{2^y}$ is equal to 2^{12}.

Choices B and C are incorrect because they are not equal to 2^{12}. Choice D is incorrect because the value of $\frac{8^x}{2^y}$ can be determined.