3.2 Exponential & Logistic Modeling

Target 3F: Model real world situations and use regressions with the use of functions

Review of Prior Concepts

The population of country A was 40 million in the year 2000 and has grown continually in the years following. The population P, in millions, of the country t years after 2000 can be modeled by the function $P(t) = 40e^{0.027t}$, where $t \geq 0$.

Based on the model, the solution to the equation $50 = 40e^{0.027t}$ gives the number of years it will take for the population of country A to reach 50 million. What is the solution to the equation expressed as a logarithm?

$50 = 40e^{.027t}$

$\frac{50}{40} = e^{.027t}$

$\ln\left(\frac{5}{4}\right) = \ln e^{.027t}$

$\ln(1.25) = .027t$

$\frac{\ln(1.25)}{0.027} = t$

More Practice

Introduction to Exponential Functions

http://www.virtualnerd.com/algebra-2/exponential-logarithmic-functions/exponentials/exponential-functions/function-definition

https://www.khanacademy.org/math/algebra/introduction-to-exponential-functions/exponential-growth-and-decay/v/exponential-growth-functions

https://www.youtube.com/watch?v=jnOwrj8OvYI

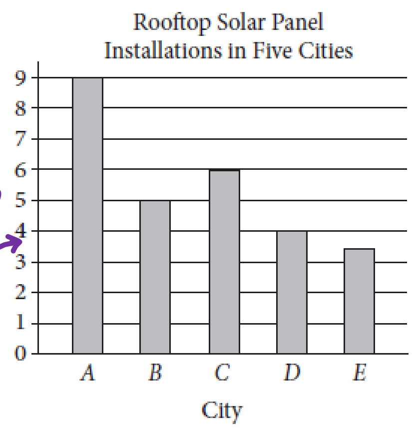
SAT Connection

Problem Solving and Data Analysis

5. Use the relationship between two variables to investigate key features of the graph.

Example:

9000
6000
5000
4000
3500
27500



A) Number of installations (in tens)
B) Number of installations (in hundreds)
C) Number of installations (in thousands)
D) Number of installations (in tens of thousands)

Total # = 27500

$A+B+C+D+E = 9+5+6+4+3.5$

$= 27.5$

$\frac{27500}{27.5} = 1000$

The number of rooftops with solar panel installations in 5 cities is shown in the graph above. If the total number of installations is 27,500, what is an appropriate label for the vertical axis of the graph?

Solution

Exponential Population Model

$$P(t) = P_0(1+r)^t$$

time

rate, percentage rate expressed as a decimal

population @ time, t

growth factor

initial population (starting) "P sub nought"

Half-Life/Radioactive Decay Model

$$y = y_0\left(\frac{1}{2}\right)^{t/t_{1/2}}$$

time

$t_{1/2}$ "t sub $\frac{1}{2}$" $\frac{1}{2}$-life time

remaining substance

initial substance (starting)

Example 1:

Write the exponential function that satisifies the given conditions:

a) initial value = \$10, increasing at a rate of 3% per day

$P(t) = P_0(1+r)^t$

$P(t) = 10(1+.03)^t$

$\boxed{P(t) = 10(1.03)^t}$

b) initial value = \$10, decreasing at a rate of 3% per day

$P(t) = P_0(1+r)^t$

$P(t) = 10(1+(-.03))^t$

$= 10(1-.03)^t$

$\boxed{P(t) = 10(.97)^t}$

c) initial population = 250, halving every 2 hours

$y = y_0\left(\frac{1}{2}\right)^{t/t_{1/2}}$

$\boxed{y = 250\left(\frac{1}{2}\right)^{t/2}}$

or $\boldsymbol{y(t) = 250\left(\frac{1}{2}\right)^{\frac{t}{2}}}$

Example 2:

2. The population of River City in the year 1910 was 4200. $\to P_0$ Assume the population increased at a rate of 2.25% per year.

a) Estimate the population in 1930 and 1900.

20 years apart

use $t=20$

use $t=-10$

$$P(t) = 4200(1+.0225)^t$$

$$P(t) = 4200(1.0225)^t$$

$$P(20) = 4200(1.0225)^{20}$$

$$= 6554.139$$

pop. in 1930 $= \boxed{6554 \text{ people}}$

$$P(-10) = 4200(1.0225)^{-10}$$

$$= 3362.143$$

pop. in 1900 $= \boxed{3362 \text{ people}}$

b) Determine when the population reached 20,000.

$$P(t) = 4200(1.0225)^t$$

$$20000 = 4200(1.0225)^t$$

$$\frac{20000}{4200} = (1.0225)^t$$

$$\ln\left(\frac{100}{21}\right) = \ln(1.0225)^t$$

$$\ln\left(\frac{100}{21}\right) = t \cdot \ln(1.0225)$$

$$\frac{\ln\left(\frac{100}{21}\right)}{\ln(1.0225)} = t$$

$$t = 70.140$$

$\boxed{\therefore \text{, population reaches 20,000 in} \approx 1980}$

3. The half-life of a certain radioactive substance is 65 days. There are 3.5g present initially.

$t_{1/2}$

a) Express the amount of substance remaining as a function of time t.

$$y = y_0\left(\frac{1}{2}\right)^{t/t_{1/2}}$$

$$\boxed{y = 3.5\left(\frac{1}{2}\right)^{t/65}}$$

or $y(t) = 3.5\left(\frac{1}{2}\right)^{\frac{t}{65}}$

b) When will there be less than 1g remaining?

$$1 = 3.5\left(\frac{1}{2}\right)^{t/65}$$

$$\frac{1}{3.5} = \left(\frac{1}{2}\right)^{t/65}$$

$$\ln\left(\frac{1}{3.5}\right) = \ln\left(\frac{1}{2}\right)^{t/65}$$

$$\ln\left(\frac{1}{3.5}\right) = \frac{t}{65}\ln\left(\frac{1}{2}\right)$$

$$\frac{\ln\left(\frac{1}{3.5}\right)}{\ln\left(\frac{1}{2}\right)} = \frac{t}{65}$$

$$65\left(\frac{\ln\left(\frac{1}{3.5}\right)}{\ln\left(\frac{1}{2}\right)}\right) = t \longrightarrow t = 117.478$$

$\boxed{117 \text{ days}}$

There will be less than 1g remaining after approximately 117 days.

More Practice

Exponential Population Models

http://www.mathsisfun.com/money/compound-interest.html

http://www.coolmath.com/algebra/17-exponentials-logarithms/03-compound-interest-01

https://youtu.be/m5Tf6vgoJtQ

Half-Life Models

http://www.coolmath.com/algebra/17-exponentials-logarithms/13-radioactive-decay-decibel-levels-01

https://youtu.be/kaxfCiP9d0w

Homework Assignment

p.270 #1,3,12,18,19,29,33,39,43

SAT Connection

Solution

Choice C is correct. Let x represent the number of installations that each unit on the y-axis represents. Then $9x$, $5x$, $6x$, $4x$, and $3.5x$ are the number of rooftops with solar panel installations in cities A, B, C, D, and E, respectively. Since the total number of rooftops is 27,500, it follows that $9x + 5x + 6x + 4x + 3.5x = 27{,}500$, which simplifies to $27.5x = 27{,}500$. Thus, $x = 1{,}000$. Therefore, an appropriate label for the y-axis is "Number of installations (in thousands)."

Choices A, B, and D are incorrect and may result from errors when setting up and calculating the units for the y-axis.