3.2 Exponential & Logistic Modeling

Target 3F: Model real world situations and use regressions with the use of functions

Review of Prior Concepts

The population (P) of a city can be represented in an equation $P = 3000e^{kt}$, where $t = 0$ represents the year 1900. In 1850, the population was 1100. Find the value of k and use this value of k to estimate the population in the year 2012.

$P = 3000e^{kt}$

$1100 = 3000e^{k(-50)}$

$\frac{1100}{3000} = e^{-50k}$

$\ln\left(\frac{11}{30}\right) = \ln e^{-50k}$

$\ln\left(\frac{11}{30}\right) = -50k$

$\frac{\ln\left(\frac{11}{30}\right)}{-50} = k$

$\boxed{.020 = k}$

Store in calculator

$P = 3000e^{.020t}$

$P(112) = 3000e^{.020(112)}$

$= 28389.206$

$\boxed{28389 \text{ people in } 2012}$

$t = 0 \rightarrow 1900$
$t = -50 \rightarrow 1850$
$k = ?$
then, when $t = 112 \rightarrow 2012$,
pop = ?

More Practice

Population Modeling

http://www.coolmath.com/algebra/17-exponentials-logarithms/06-population-exponential-growth-01
http://www.purplemath.com/modules/expoprob2.htm
https://www.youtube.com/watch?v=63udRYh04sY

SAT Connection

Passport to Advanced Mathematics

1. Create a quadratic or exponential function or equation that models a context.

Example: A radioactive substance decays at an annual rate of 13 percent. If the initial amount of the substance is 325 grams, which of the following functions f models the remaining amount of the substance, in grams, t years later?

$y_0 = 325$

A) $f(t) = 325(0.87)^t$

B) $f(t) = 325(0.13)^t$

C) $f(t) = 0.87(325)^t$

D) $f(t) = 0.13(325)^t$

$1 - .13$

$.87$

$y = y_0(1+r)^t$

$y = 325(.87)^t$

Solution

Logistic Model

$$f(x) = \frac{c}{1 + a \cdot b^x}$$

- Limiting factor
- Limit to growth
- maximum sustainable population
- carrying capacity

Example 1:

p. 271

#24

$f(x) = \frac{c}{1+a \cdot b^x}$

Limit to growth = 60

$f(x) = \frac{60}{1+a \cdot b^x}$

initial value = 12

when $x = 0$, $y = 12$

$12 = \frac{60}{1+a \cdot b^0}$

$12 = \frac{60}{1+a}$

$(1+a)\,12 = \frac{60}{\cancel{1+a}}\cancel{(1+a)}$

$12 + 12a = 60$

$12a = 48$

$a = 4$

$f(x) = \frac{60}{1+4 \cdot b^x}$

pass thru (1,24)

$24 = \frac{60}{1+4 \cdot b^1}$

$24 = \frac{60}{1+4b}$

$(1+4b)\,24 = \frac{60}{\cancel{1+4b}}\cancel{(1+4b)}$

$24 + 96b = 60$

$96b = 36$

$b = 3/8$

$$\boxed{f(x) = \frac{60}{1+4(3/8)^x}}$$

Example 2:
p.271 #26

$f(x) = \frac{c}{1+a\cdot b^x}$ limit to growth = 30

$f(x) = \frac{30}{1+a\cdot b^x}$

initial value = 5
when x = 0, y = 5

$5 = \frac{30}{1+a\cdot b^0}$

$5 = \frac{30}{1+a}$

$(1+a)\,5 = \frac{30}{\cancel{1+a}}\cancel{(1+a)}$

$5+5a = 30$

$5a = 25$

$a = 5$

pass thru (3,15)

$f(x) = \frac{30}{1+5\cdot b^x}$

$15 = \frac{30}{1+5\cdot b^3}$

$15 = \frac{30}{1+5b^3}$

$(1+5b^3)\,15 = \frac{30}{\cancel{1+5b^3}}\cancel{(1+5b^3)}$

$15+75b^3 = 30$

$75b^3 = 15$

$b^3 = 15/75$

$\sqrt[3]{b^3} = \sqrt[3]{\frac{1}{5}}$

$b = 0.585$

$$\boxed{f(x) = \frac{30}{1+5(0.585)^x}}$$

Example 3:
p.272 #46

a) initial population = ?

$t = 0$

$P(0) = \frac{1001}{1+90e^{-.2(0)}}$

$= \frac{1001}{1+90e^0}$ $e^0 = 1$

$= \frac{1001}{1+90}$

$= \frac{1001}{91} = \boxed{11 \text{ deer}}$

b) # of deer = 600

$600 = \frac{1001}{1+90e^{-.2t}}$

$(1+90e^{-.2t})\,600 = \frac{1001}{\cancel{1+90e^{-.2t}}}\cancel{(1+90e^{-.2t})}$

$600 + 54000e^{-.2t} = 1001$

$54000e^{-.2t} = 401$

$e^{-.2t} = \frac{401}{54000}$

$\ln e^{-.2t} = \ln\left(\frac{401}{54000}\right)$

$-.2t = \ln\left(\frac{401}{54000}\right)$

$t = \frac{\ln\left(\frac{401}{54000}\right)}{-.2}$

$t = 24.514$

$\boxed{24.514 \text{ years}}$

c) maximum # of deer
(maximum sustainable population)

$\boxed{1001 \text{ deer}}$

More Practice

Logistic Models
http://www.ck12.org/book/CK-12-Precalculus-Concepts/section/3.7/
https://www.youtube.com/watch?v=LyJrUtzKtwI
https://www.youtube.com/watch?v=OSMPeY354cU

Homework Assignment

p.271 #23,28,45,47,50

SAT Connection
Solution

Choice A is correct. Each year, the amount of the radioactive substance is reduced by 13 percent from the prior year's amount; that is, each year, 87 percent of the previous year's amount remains. Since the initial amount of the radioactive substance was 325 grams, after 1 year, 325(0.87) grams remains; after 2 years $325(0.87)(0.87) = 325(0.87)^2$ grams remains; and after t years, $325(0.87)^t$ grams remains. Therefore, the function $f(t) = 325(0.87)^t$ models the remaining amount of the substance, in grams, after t years.

Choice B is incorrect and may result from confusing the amount of the substance remaining with the decay rate. Choices C and D are incorrect and may result from confusing the original amount of the substance and the decay rate.