DATE: ______________

Unit 3 (Chapter 3): Exponential, Logistic, & Logarithmic Functions **Pre-Calculus**

3.3 Logarithmic Functions & Their Graphs

Target 3B: Know and understand the inverse relationships of exponential and logarithmic equations

SAT Connection

Passport to Advanced Math

14. Use structure to isolate or identify a quantity of interest in an expression

Example: Jessica opened a bank account that earns 2 percent interest compounded annually. Her initial deposit was \$100, and she uses the expression $\$100(x)^t$ to find the value of the account after t years.
What is the value of x in the expression?

$\therefore,\ x = 1.02$

1 year
$\$100 + 2\%(100)$
$\$100 + .02(100)$
$100(1 + .02)$
$100(1.02)$

2 years
$100(1.02) + 2\%(100(1.02))$
$100(1.02) + .02(100(1.02))$
$100(1.02)(1 + .02)$
$100(1.02)(1.02)$
$100(1.02)^2$

3 years → $100(1.02)^3$
4 years → $100(1.02)^4$
t years → $100(1.02)^t$

Solution

Logarithmic Functions

Logarithmic Form

$\log_b a = r$

base, argument, result

"Log base b of a equals r"

Exponential Form

$b^r = a$

base, result, argument

*Logarithmic & Exponential Functions are Inverses! :)

Examples

Evaluate the logarithmic expression.

1. $\log_3 9 = r$

$3^r = 9$
$3^r = 3^2$
$r = 2$
$\log_3 9 = \boxed{2}$

same base, exponents = :)

2. $\log_3 \frac{1}{27} = r$

$3^r = \frac{1}{27}$
$3^r = \frac{1}{3^3}$
$3^r = 3^{-3}$
$r = -3$
$\log_3 \frac{1}{27} = \boxed{-3}$

3. $\log_2 \sqrt{8} = r$

$2^r = \sqrt{8}$
$2^r = \sqrt{2^3}$
$2^r = (2^3)^{1/2}$
$2^r = 2^{3/2}$
$r = 3/2$
$\log_2 \sqrt{8} = \boxed{\frac{3}{2}}$

4. $\ln e^2 = r$

base = e

$e^r = e^2$
$r = 2$
$\ln e^2 = \boxed{2}$

Common Log → has base 10 $\boxed{\log}$

$\log x = \log_{10} x$

Natural Log → has base e $\boxed{\ln}$ "el" "en"

$\ln x = \log_e x$

Examples

Using a calculator, evaluate the logarithmic expression.

1. $\log 4$

.602

2. $\ln 2$

.693

3. $\log_2 5$

2.322

Properties of Logs/Natural Logs

$\log_b 1 = \underline{0}$ $b^{0} = 1$ $e^{0} = 1$ $\ln 1 = \underline{0}$

$\log_b b = \underline{1}$ $b^{1} = b$ $e^{1} = e$ $\ln e = \underline{1}$

$\log_b b^y = \underline{y}$ $b^{y} = b^y$ $e^{y} = e^y$ $\ln e^y = \underline{y}$

$b^{\log_b y} = \underline{y}$ $\log_b y = \log_b y$ $\ln y = \ln y$ $e^{\ln y} = \underline{y}$

Using the properties of logarithms, evaluate the logarithmic expression.

1. $\log_3 9$

$\log_3 3^2$

$\boxed{2}$

2. $\log_5 125$

$\log_5 5^3$

$\boxed{3}$

3. $e^{\ln 4}$

$\boxed{4}$

4. $5^{\log_5 8}$

$\boxed{8}$

5. $\log_4 1$

$\boxed{0}$

6. $\log_{10} \frac{1}{100}$

$\log_{10} \frac{1}{10^2}$

$\log_{10} 10^{-2}$

$\boxed{-2}$

7. $\ln e^8$

$\boxed{8}$

8. $x^{\log_x 7}$

$\boxed{7}$

Solve the equation for *x*.

9. $\log x = 5$

$\log_{10} x = 5$

$10^{\log_{10} x} = 10^5$

$x = 10^5$

$\boxed{x = 100{,}000}$

10. $\frac{2}{2}\log x = \frac{-6}{2}$

$\log x = -3$

$\log_{10} x = -3$

$10^{\log_{10} x} = 10^{-3}$

$x = 10^{-3}$

$x = \frac{1}{10^3}$

$\boxed{x = \frac{1}{1000}}$

11. $\ln x^2 = 4$

$e^{\ln x^2} = e^4$

$x^2 = e^4$

$\sqrt{x^2} = \pm\sqrt{e^4}$

$x = \pm\sqrt{e^4}$

$x = \pm(e^4)^{1/2}$

$\boxed{x = \pm e^2}$

More Practice

Logarthims

https://www.khanacademy.org/math/algebra2/exponential-and-logarithmic-functions/introduction-to-logarithms/a/intro-to-logarithms
http://www.themathpage.com/aprecalc/logarithmic-exponential-functions.htm
http://www.sosmath.com/algebra/logs/log4/log41/log41.html
http://www.regentsprep.org/regents/math/algtrig/ATE9/logs.htm
https://youtu.be/Z5myJ8dg_rM

Homework Assignment

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SAT Connection

Solution

The correct answer is 1.02. The initial deposit earns 2 percent interest compounded annually. Thus at the end of 1 year, the new value of the account is the initial deposit of \$100 plus 2 percent of the initial deposit: $\$100 + \frac{2}{100}(\$100) = \$100(1.02)$. Since the interest is compounded annually, the value at the end of each succeeding year is the sum of the previous year's value plus 2 percent of the previous year's value. This is again equivalent to multiplying the previous year's value by 1.02. Thus, after 2 years, the value will be $\$100(1.02)(1.02) = \$100(1.02)^2$; after 3 years, the value will be $\$100(1.02)^3$; and after t years, the value will be $\$100(1.02)^t$. Therefore, in the formula for the value for Jessica's account after t years, $\$100(x)^t$, the value of x must be 1.02.