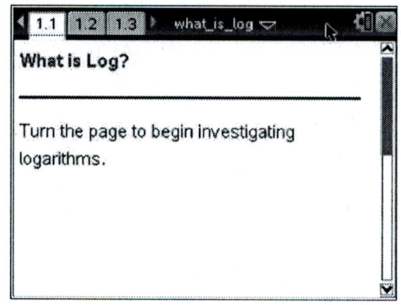
What is Log?
Student Activity

Date ______________



Open the TI-Nspire document *What_is_Log.tns.*

You may have noticed that above [10^x] is [log]. What does *log* mean? Why is [log] placed above the exponential key? You will investigate these questions in this activity.

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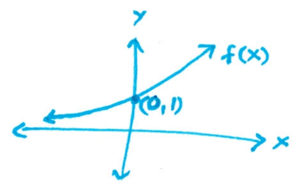
Press [ctrl] ▶ and [ctrl] ◀ to navigate through the lesson.

1. The graph of the function $f(x) = 2^x$ is shown.

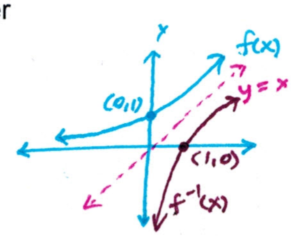
   a. What are the domain and range of $f(x)$?

   Domain : $(-\infty, \infty)$
   Range : $(0, \infty)$ Zero not included



   b. Recall that $f(x) = 2^x$ is a one-to-one function, so it has an inverse reflected over the line $y = x$. What are the domain and range of $f^{-1}(x)$?

   Domain : $(0, \infty)$
   Range : $(-\infty, \infty)$ Zero not included



   c. Point P is a point on $f(x)$. Move the Show Reflection slider to *Yes* and then move point P. As you do so, point P' invisibly traces the graph of $f^{-1}(x)$. Since $f(x)$ can be written as $y = 2^x$, write a corresponding equation for the inverse.

   $f(x) = 2^x$
   $y = 2^x$
   we switch the x and y.

   Corresponding Equation
   $x = 2^y$

   d. The equation $x = 2^y$ cannot be written as a function of y in terms of x without new notation. Move the Show Function slider to *Yes*. The inverse of $f(x)$ is actually $f^{-1}(x) = \log_2(x)$. In general, $\log_b x = y$ is equivalent to $b^y = x$ for $x > 0$, $b > 0$ and $b \neq 1$. Why do you think x and b must be greater than 0? Why can b not be equal to 1?

   $x = 2^y \iff f^{-1}(x) = \log_2 x$

   $y = \log_b x$

   - x must be greater than zero $(x>0)$ b/c domain is $(0, \infty)$
   - b must be greater than zero $(b>0)$ b/c negative values of b result in negative values of x, and x must be greater than 0.
   - b can't be 1 since b=1 would result in a linear function.

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e. Move point P so that its coordinates are $(1, 2)$. The point (1, 2) on $f(x) = 2^x$ indicates that $2^1 = 2$. P' has the coordinates $(2, 1)$. The point (2, 1) on $f^{-1}(x) = \log_2(x)$ indicates that $\log_2 2 = 1$. Use this relationship between exponential expressions and logarithmic expressions to complete the following table. (Move point P as necessary.)

P	P'	Exponential Expression	Logarithmic Expression
(1, 2)	(2, 1)	$2^1 = 2$	$\log_2 2 = 1$
(2, 4)	(4, 2)	$2^2 = 4$	$\log_2 4 = 2$
(3, 8)	(8, 3)	$2^3 = 8$	$\log_2 8 = 3$
(0, 1)	(1, 0)	$2^0 = 1$	$\log_2 1 = 0$
$(-1, \frac{1}{2})$	$(\frac{1}{2}, -1)$	$2^{-1} = \frac{1}{2^1} = \frac{1}{2}$	$\log_2 \frac{1}{2} = -1$
$\left(-2, \frac{1}{4}\right)$	$(\frac{1}{4}, -2)$	$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$	$\log_2 \frac{1}{4} = -2$
$(-3, \frac{1}{8})$	$(\frac{1}{8}, -3)$	$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$	$\log_2 \frac{1}{8} = -3$

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2. Solve the logarithmic equation $\log_2 32 = y$ using the patterns from question 1. Then, use the slider to change the n-value to solve the logarithmic equation. How does the exponential equation verify your result?

$\log_2 32 = n \iff \underbrace{2^n = 32}_{\text{exponential equation}}$ $\therefore$ $n = 5$ since $2^5 = 32$.

base base

3. Solve the equation $\log_4 \frac{1}{256} = y$.

$\log_4 \frac{1}{256} = n \iff \underbrace{4^n = \frac{1}{256}}_{\text{exponentia equation}}$ $\therefore$ $n = -4$ since $4^{-4} = \frac{1}{4^4} = \frac{1}{256}$.

base base