

3.4 Properties of Logarithmic Functions

Target 3C: Understand properties of common and natural logarithmic functions
Target 3E: Know and apply product, quotient and power rules of logarithmic functions



SAT Connection

Passport to Advanced Math

8. Solve a system of one linear equation and one quadratic equation.

Example:

If $\frac{x^{a^2}}{x^{b^2}} = x^{16}$, $x > 1$, and $a + b = 2$, what is the value

of $a - b$?

- A) 8
- B) 14
- C) 16
- D) 18

Solution

Properties of Logs/Natural Logs

Product Property: $\log_b(xy) =$

$$\ln(xy) =$$



Quotient Property: $\log_b\left(\frac{x}{y}\right) =$

$$\ln\left(\frac{x}{y}\right) =$$

Power Property: $\log_b x^c =$

$$\ln x^c =$$

Change of Base: $\log_b x = \frac{\log_a x}{\log_a b}$

$$\log_b x = \frac{\ln x}{\ln b}$$

Examples

Using the properties of logarithms, expand the logarithmic expression.

1. $\ln 3x$

2. $\log\left(\frac{4x}{y^2}\right)$

3. $\log_2(25x^3)$

4. $\log\sqrt[3]{\frac{x^2}{y}}$

Using the properties of logarithms, condense the logarithms into a single expression.

5. $\log x + 3 \log y$

6. $\ln 4x - \ln 2y$

7. $2\left[\log x - \frac{1}{6}\log y + 4\log(a - 1)\right]$

Write the expression as a natural logarithm.

8. $\log_5 x$

9. $\log_4(2x + y)$

More Practice

Properties of Logarithms

<https://www.khanacademy.org/math/algebra2/exponential-and-logarithmic-functions/properties-of-logarithms/v/introduction-to-logarithm-properties>

http://www.algebralab.org/lessons/lesson.aspx?file=algebra_logarithmproperties.xml

<http://www.regentsprep.org/regents/math/algtrig/ate9/LogPrac.htm>

<http://www.mathguide.com/lessons2/Logs.html>

<https://www.youtube.com/watch?v=SxF44olWTyk>

<https://www.youtube.com/watch?v=eLapHtvQbFo>

Homework Assignment

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SAT Connection
Solution

Choice A is correct. Since the numerator and denominator of $\frac{x^{a^2}}{x^{b^2}}$ have a common base, it follows by the laws of exponents that this expression can be rewritten as $x^{a^2 - b^2}$. Thus, the equation $\frac{x^{a^2}}{x^{b^2}} = 16$ can be rewritten as $x^{a^2 - b^2} = x^{16}$. Because the equivalent expressions have the common base x , and $x > 1$, it follows that the exponents of the two expressions must also be equivalent. Hence, the equation $a^2 - b^2 = 16$ must be true. The left-hand side of this new equation is a difference of squares, and so it can be factored: $(a + b)(a - b) = 16$. It is given that $(a + b) = 2$; substituting 2 for the factor $(a + b)$ gives $2(a - b) = 16$. Finally, dividing both sides of $2(a - b) = 16$ by 2 gives $a - b = 8$.

Choices B, C, and D are incorrect and may result from errors in applying the laws of exponents or errors in solving the equation $a^2 - b^2 = 16$.