3.4 Properties of Logarithmic Functions

Target 3C: Understand properties of common and natural logarithmic functions

Target 3E: Know and apply product, quotient and power rules of logarithmic functions

SAT Connection

Passport to Advanced Math

8. Solve a system of one linear equation and one quadratic equation.

Example:

If $\dfrac{x^{a^2}}{x^{b^2}} = x^{16}$, $x > 1$, and $a + b = 2$, what is the value of $a - b$?

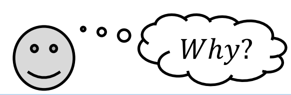
A) 8

B) 14

C) 16

D) 18

Solution



Properties of Logs/Natural Logs

Product Property: $\log_b(xy) = \log_b x + \log_b y$

$\ln(xy) = \ln x + \ln y$

$\log_b(xy) = \log_b x + \log_b y$

$b^{\log_b(xy)} = b^{\log_b x + \log_b y}$

$xy = b^{\log_b x} \cdot b^{\log_b y}$

✓ $\uparrow = x \cdot y$

$x^2 \cdot x^3 = x^{2+3} = x^5$

Quotient Property: $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

$\ln\left(\frac{x}{y}\right) = \ln x - \ln y$

$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

$b^{\log_b\left(\frac{x}{y}\right)} = b^{\log_b x - \log_b y}$

$\frac{x}{y} = b^{\log_b x} / b^{\log_b y}$

$\uparrow = \frac{x}{y}$

$\frac{x^5}{x^3} = x^{5-3} = x^2$

Power Property: $\log_b x^c = c \log_b x$

$\ln x^c = c \ln x$

Let $m = \log_b x$, then

$b^m = x$

$(b^m)^c = x^c$

$b^{mc} = x^c \rightarrow \log_b x^c = mc$

So, $\log_b x^c = (\log_b x) \cdot c$

Change of Base: $\log_b x = \dfrac{\log_a x}{\log_a b}$

$\log_b x = \dfrac{\ln x}{\ln b}$

Let $\log_b x = r$, then

$b^r = x$

$\log_a b^r = \log_a x$

$r \log_a b = \log_a x \longrightarrow r = \dfrac{\log_a x}{\log_a b}$

Examples

Using the properties of logarithms, expand the logarithmic expression.

1. $\ln 3x$

$\ln(3\cdot x)$ *product

$\boxed{\ln 3 + \ln x}$

2. $\log\left(\frac{4x}{y^2}\right)$

$\log(4x) - \log y^2$ *quotient

$\log 4 + \log x - \log y^2$ *product

$\log 2^2 + \log x - \log y^2$

$\boxed{2\log 2 + \log x - 2\log y}$ *power

3. $\log_2 25x^3$

$\log_2 25 + \log_2 x^3$ *product

$\log_2 5^2 + \log_2 x^3$

$\boxed{2\log_2 5 + 3\log_2 x}$ *power

4. $\log\sqrt[3]{\frac{x^2}{y}}$

$\log\left(\frac{x^2}{y}\right)^{1/3}$ *rewrite + simplify

$\log\left(\frac{x^{2/3}}{y^{1/3}}\right)$

$\log x^{2/3} - \log y^{1/3}$ *quotient

$\boxed{\frac{2}{3}\log x - \frac{1}{3}\log y}$ *power

Using the properties of logarithms, condense the logarithms into a single expression.

5. $\log x + 3\log y$

$\log x + \log y^3$ *power

$\boxed{\log(xy^3)}$ *product

6. $\ln 4x - \ln 2y$

$\ln\left(\frac{4x}{2y}\right)$ *quotient

$\boxed{\ln\left(\frac{2x}{y}\right)}$ *simplify

7. $2\log x - \frac{1}{3}\log y + \log a$

$\log x^2 - \log y^{1/3} + \log a$ *power

$\log\left(\frac{x^2}{y^{1/3}}\right) + \log a$ *quotient

$\boxed{\log\left(\frac{ax^2}{y^{1/3}}\right)}$ *product

Write the expression as a natural logarithm.

8. $\log_5 x$

$\boxed{\frac{\ln x}{\ln 5}}$ *change of base

9. $\log_4(2x+y)$

$\boxed{\frac{\ln(2x+y)}{\ln 4}}$ *change of base

More Practice

Properties of Logarthims

https://www.khanacademy.org/math/algebra2/exponential-and-logarithmic-functions/properties-of-logarithms/v/introduction-to-logarithm-properties
http://www.algebralab.org/lessons/lesson.aspx?file=algebra_logarithmproperties.xml
http://www.regentsprep.org/regents/math/algtrig/ate9/LogPrac.htm
http://www.mathguide.com/lessons2/Logs.html
https://www.youtube.com/watch?v=SxF44olWTyk
https://www.youtube.com/watch?v=eLapHtvQbFo

Homework Assignment

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SAT Connection

Solution

Choice A is correct. Since the numerator and denominator of $\frac{x^{a^2}}{x^{b^2}}$ have a common base, it follows by the laws of exponents that this expression can be rewritten as $x^{a^2 - b^2}$. Thus, the equation $\frac{x^{a^2}}{x^{b^2}} = 16$ can be rewritten as $x^{a^2 - b^2} = x^{16}$. Because the equivalent expressions have the common base x, and $x > 1$, it follows that the exponents of the two expressions must also be equivalent. Hence, the equation $a^2 - b^2 = 16$ must be true. The left-hand side of this new equation is a difference of squares, and so it can be factored: $(a + b)(a - b) = 16$. It is given that $(a + b) = 2$; substituting 2 for the factor $(a + b)$ gives $2(a - b) = 16$. Finally, dividing both sides of $2(a - b) = 16$ by 2 gives $a - b = 8$.

Choices B, C, and D are incorrect and may result from errors in applying the laws of exponents or errors in solving the equation $a^2 - b^2 = 16$.