3.5 Equation Solving & Modeling

Target 3B: Know and understand the inverse relationships of exponential and logarithmic equations

SAT Connection
Problem Solving and Data Analysis
4. Create an equivalent form of an algebraic expression

Example:

$$9a^4 + 12a^2b^2 + 4b^4$$

Which of the following is equivalent to the expression shown above?

A) $(3a^2 + 2b^2)^2$

B) $(3a + 2b)^4$

C) $(9a^2 + 4b^2)^2$

D) $(9a + 4b)^4$

$9a^4 + 12a^2b^2 + 4b^4$
$= (\underbrace{3a^2}_{x})^2 + 2(\underbrace{3a^2}_{x})(\underbrace{2b^2}_{y}) + (\underbrace{2b^2}_{y})^2$
$= x^2 + 2xy + y^2$
$= (x+y)(x+y)$
$= (3a^2 + 2b^2)(3a^2 + 2b^2)$
$= (3a^2 + 2b^2)^2$

Let $x = 3a^2$
Let $y = 2b^2$

OR distribute & check:
$(3a^2 + 2b^2)^2$
$= (3a^2 + 2b^2)(3a^2 + 2b^2)$
$= 9a^4 + 6a^2b^2 + 6a^2b^2 + 4b^4$
$= 9a^4 + 12a^2b^2 + 4b^4$

Solution

Orders of Magnitude and Logarithmic Models

Explain in your own words what **Order of Magnitude** means and give an example.

p. 294-295

answers vary:
Orders of Magnitude → common log of a positive quantity
→ powers of 10

ex: kilometer → 1000 meters
10^3 meters
∴, km is 3 orders of magnitude longer than a meter

Read through *Example 5*, then find the answer to the following problem:

In January of 2010, the country of Haiti was hit by a disastrous 7.0 magnitude earthquake. In February of 2010, a 3.8 magnitude earthquake was recorded 45 miles northwest of Chicago. How many times stronger was the Haiti earthquake than the Illinois earthquake?

$7.0 - 3.8 = 3.2$

$10^{3.2} = 1584.893$

Haiti Earthquake was approximately 1600 times stronger than Chicago Earthquake.

Properties of Logarithms Extra Practice

1. Expand using properties of logarithms:

a) $\log_3 rt$

$\boxed{\log_3 r + \log_3 t}$ *product

b) $\log_f k^3$

$\boxed{3\log_f k}$ *power

c) $\log_5 2f^3h^4$

$\log_5 2 + \log_5 f^3 + \log_5 h^4$ *product

$\boxed{\log_5 2 + 3\log_5 f + 4\log_5 h}$ *power

d) $\ln \frac{u}{7}$

$\boxed{\ln u - \ln 7}$ *quotient

e) $\log_4 \frac{3y}{gh}$

$\log_4(3y) - \log_4(gh)$ *quotient

$\log_4 3 + \log_4 y - (\log_4 g + \log_4 h)$ *product

$\boxed{\log_4 3 + \log_4 y - \log_4 g - \log_4 h}$

f) $\log_9 \frac{2d}{5w^3}$

$\log_9(2d) - \log_9(5w^3)$ *quotient

$\log_9 2 + \log_9 d - (\log_9 5 + \log_9 w^3)$ *product *power

$\boxed{\log_9 2 + \log_9 d - \log_9 5 - 3\log_9 w}$

2. Write as a single logarithm using properties of logarithms:

a) $\log_2 t + \log_2 6 + \log_2 k$

$\log_2(t\cdot 6\cdot k)$

$\boxed{\log_2 6tk}$

b) $2\log_4 m + 5\log_4 n + \log_4 k$

$\log_4 m^2 + \log_4 n^5 + \log_4 k$

$\boxed{\log_4 m^2n^5k}$

c) $\frac{1}{2}\log_8 a + \frac{1}{3}\log_8 b$

$\log_8 a^{1/2} + \log_8 b^{1/3}$

$\boxed{\log_8 a^{1/2}b^{1/3}}$

or

$\boxed{\log_8 \sqrt{a}\sqrt[3]{b}}$

d) $\log_3 y - \log_3 6 - 2\log_3 t$

$\log_3 y - \log_3 6 - \log_3 t^2$

$\log_3\left(\frac{y}{6}\right) - \log_3(t^2)$

$\boxed{\log_3\left(\frac{y}{6t^2}\right)}$

e) $2\log_6 t + 3\log_6 t + 5\log_6 t$

$\log_6 t^2 + \log_6 t^3 + \log_6 t^5$ or $10\log_6 t$

$\log_6(t^2\cdot t^3\cdot t^5)$

$\boxed{\log_6(t^{10})}$

$\log_6 t^{10}$

f) $\ln x - 3\ln x + 2\ln x$

$\ln x - \ln x^3 + \ln x^2$ or $0\ln x$

$\ln\left(\frac{x}{x^3}\right) + \ln x^2$

0

$\ln\left(\frac{x\cdot x^2}{x^3}\right)$

$\ln\left(\frac{x^3}{x^3}\right)$

$\ln 1$

$\boxed{0}$

More Practice

Orders of Magnitude
https://www.khanacademy.org/math/pre-algebra/pre-algebra-exponents-radicals/pre-algebra-orders-of-magnitude/v/orders-of-magnitude-exercise-example-1

Properties of Logarithms
https://www.khanacademy.org/math/algebra2/exponential-and-logarithmic-functions/properties-of-logarithms/v/introduction-to-logarithm-properties
http://www.algebralab.org/lessons/lesson.aspx?file=algebra_logarithmproperties.xml
http://www.regentsprep.org/regents/math/algtrig/ate9/LogPrac.htm
http://www.mathguide.com/lessons2/Logs.html
https://www.youtube.com/watch?v=SxF44olWTyk
https://www.youtube.com/watch?v=eLapHtvQbFo

Homework Assignment

p.301 #29,37,39,41,45,47

SAT Connection

Solution

Choice A is correct. If a polynomial expression is in the form $(x)^2 + 2(x)(y) + (y)^2$, then it is equivalent to $(x + y)^2$. Because $9a^4 + 12a^2b^2 + 4b^4 = (3a^2)^2 + 2(3a^2)(2b^2) + (2b^2)^2$, it can be rewritten as $(3a^2 + 2b^2)^2$.

Choice B is incorrect. The expression $(3a + 2b)^4$ is equivalent to the product $(3a + 2b)(3a + 2b)(3a + 2b)(3a + 2b)$. This product will contain the term $4(3a)^3\,(2b) = 216a^3b$. However, the given polynomial, $9a^4 + 12a^2b^2 + 4b^4$, does not contain the term $216a^3b$. Therefore, $9a^4 + 12a^2b^2 + 4b^4 \neq (3a + 2b)^4$. Choice C is incorrect. The expression $(9a^2 + 4b^2)^2$ is equivalent to the product $(9a^2 + 4b^2)(9a^2 + 4b^2)$. This product will contain the term $(9a^2)(9a^2) = 81a^4$. However, the given polynomial, $9a^4 + 12a^2b^2 + 4b^4$, does not contain the term $81a^4$. Therefore, $9a^4 + 12a^2b^2 + 4b^4 \neq (9a^2 + 4b^2)^2$. Choice D is incorrect. The expression $(9a + 4b)^4$ is equivalent to the product $(9a + 4b)(9a + 4b)(9a + 4b)\,(9a + 4b)$. This product will contain the term $(9a)(9a)(9a)(9a) = 6{,}561a^4$. However, the given polynomial, $9a^4 + 12a^2b^2 + 4b^4$, does not contain the term $6{,}561a^4$. Therefore, $9a^4 + 12a^2b^2 + 4b^4 \neq (9a + 4b)^4$.