

3.7 Quadratic Systems Notes

Target 7A. Solve a system of various functions using technology.

By drawing a second parabola in the Solve It, you created a quadratic system.
Essential Understanding You can solve systems involving quadratic equations using methods similar to the ones used to solve systems of linear equations.
 The points where the graphs of the equations intersect represent the solutions of a system.



Key Concept Solutions of a Linear-Quadratic System

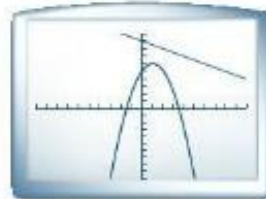
A system of one quadratic equation and one linear equation can have two solutions, one solution, or no solution.



Two solutions



One solution



No solution



Problem 1 Solving a Linear-Quadratic System by Graphing

Multiple Choice Which numbers are y-values of the solutions of the system of equations?

$$\begin{cases} y = -x^2 + 5x + 6 \\ y = x + 6 \end{cases}$$

- (A) 4 only (B) 6 only (C) 4 and 6 (D) 6 and 10

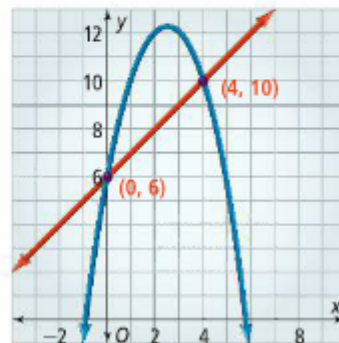
Graph the equations. Find their intersections.

The solutions appear to be (0, 6) and (4, 10).

Check

$y = -x^2 + 5x + 6$	$y = x + 6$
$6 \stackrel{?}{=} -(0)^2 + 5(0) + 6$	$6 \stackrel{?}{=} 0 + 6$
$6 = 6 \checkmark$	$6 = 6 \checkmark$
$y = -x^2 + 5x + 6$	$y = x + 6$
$10 \stackrel{?}{=} -(4)^2 + 5(4) + 6$	$10 \stackrel{?}{=} 4 + 6$
$10 = 10 \checkmark$	$10 = 10 \checkmark$

The y-values of the solutions are 6 and 10, choice D.





Problem 2 Solving a Linear-Quadratic System Using Substitution

What is the solution of the system of equations? $\begin{cases} y = -x^2 - x + 6 \\ y = x + 3 \end{cases}$

Think

Substitute $x + 3$ for y in the quadratic equation.

$$x + 3 = -x^2 - x + 6$$

Write in standard form.

$$x^2 + 2x - 3 = 0$$

Factor. Solve for x .

$$(x - 1)(x + 3) = 0 \\ x = 1 \text{ or } x = -3$$

Substitute for x in $y = x + 3$.

$$x = 1 \rightarrow y = 1 + 3 = 4 \\ x = -3 \rightarrow y = -3 + 3 = 0 \\ \text{The solutions are } (1, 4) \text{ and } (-3, 0).$$

Write

You can solve quadratic-quadratic systems using the same methods you used for linear-quadratic systems.



Problem 3 Solving a Quadratic System of Equations

What is the solution of the system? $\begin{cases} y = -x^2 - x + 12 \\ y = x^2 + 7x + 12 \end{cases}$

Method 1 Use substitution.

Substitute $y = -x^2 - x + 12$ for y in the second equation. Solve for x .

$$-x^2 - x + 12 = x^2 + 7x + 12 \quad \text{Substitute for } y.$$

$$-2x^2 - 8x = 0 \quad \text{Write in standard form.}$$

$$-2x(x + 4) = 0 \quad \text{Factor.}$$

$$x = 0 \text{ or } x = -4 \quad \text{Solve for } x.$$

Substitute each value of x into either equation. Solve for y .

$$y = x^2 + 7x + 12 \quad y = x^2 + 7x + 12$$

$$y = (0)^2 + 7(0) + 12 \quad y = (-4)^2 + 7(-4) + 12$$

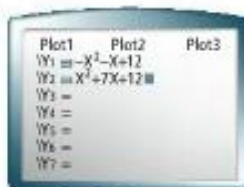
$$y = 0 + 0 + 12 = 12 \quad y = 16 - 28 + 12 = 0$$

The solutions are $(0, 12)$ and $(-4, 0)$.

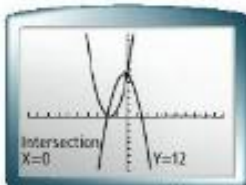
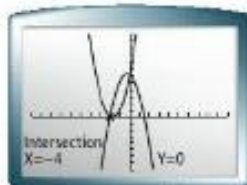
Method 2 Graph the equations.

Use a graphing calculator.

Define functions Y_1 and Y_2 .



Use the **INTERSECT** feature to find the points of intersection.

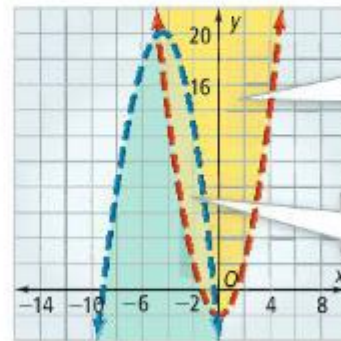
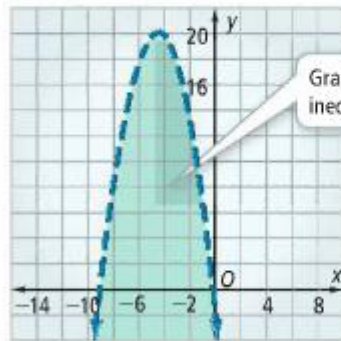


The solutions are $(-4, 0)$ and $(0, 12)$.



Problem 4 Solving a Quadratic System of Inequalities

What is the solution of the system of inequalities? $\begin{cases} y < -x^2 - 9x - 2 \\ y > x^2 - 2 \end{cases}$



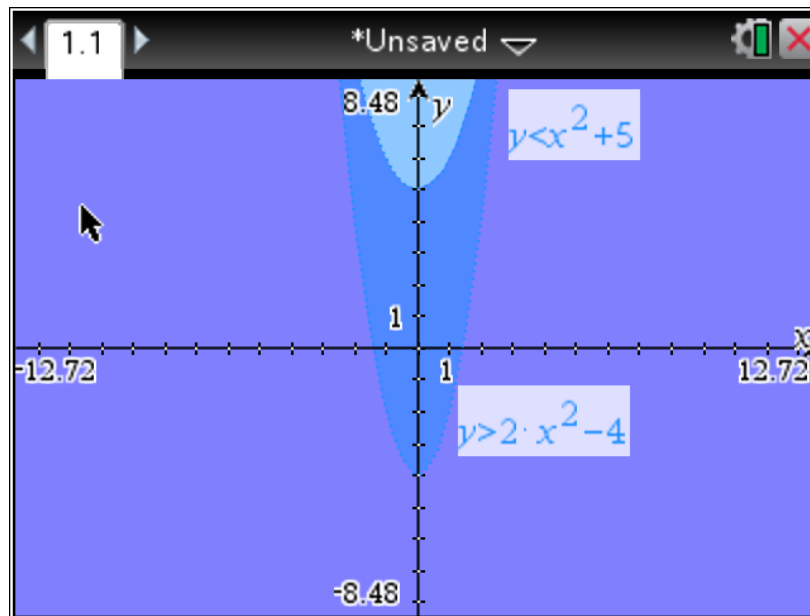
Graph the second inequality.

Identify where the graphs overlap.

The solution of this system is the region where the graphs overlap. The region contains no boundary points.

Problem 5 (we did it in class)

What is the solution of the system of inequalities? $\begin{cases} y < x^2 + 5 \\ y > 2x^2 - 4 \end{cases}$



The solution of this system is again the region where the graphs overlap. The region contains no boundary points; i.e., any point directly on the parabola is not a solution of the system of inequalities. It's also important to note that our window could have been adjusted to see two intersection points, which are $(-3, 14)$ and $(3, 14)$. Do you think this is important? If so, why? Are these points part of the solution set? Why or why not?