

Angular and Linear Speed



Imagine a merry-go-round in a school playground. A vertical steel pipe runs through the merry-go-round into the soil beneath it and serves as the axis about which it rotates.

Angular velocity (speed) represents the rate of rotation of the merry-go-round, that is, how much angle it turns through in a given amount of time.

There are two children on this merry-go-round - one is located two feet from the center, while the other is sitting at the edge or perimeter of the merry-go-round, four feet from center. Which child do you think experiences a greater angular velocity?

Linear speed – rate at which linear position changes over time (measured in units, like miles per hour)

Angular speed – rate at which angular position changes over time (measured in units, like revolutions per minute)

1 \angle of radian measure
= 1 radius of arc length

Examples:

1) If the hard drive in a computer rotates at 3600 rotations per minute. Express the angular speed of a hard drive in radians per minute. (Note: 1 revolution = 2π radians)

given \rightarrow 3600 rotations per minute
final answer \rightarrow

$$\frac{3600 \text{ revolutions}}{\text{minute}} \cdot \frac{2\pi \text{ radians}}{1 \text{ revolution}}$$

$$= 7200\pi \frac{\text{radians}}{\text{minute}}$$

$$= 22619.467 \text{ rads/min}$$

2) A windmill, used to generate electricity, has blades that are 10 feet in length. The propeller is rotating around at 4 revolutions per second. Find the linear speed, in feet per second of the tips of the blades.

given \rightarrow 4 revolutions per second
final answer

$$\frac{4 \text{ rev}}{\text{sec}} \cdot \frac{10 \text{ ft}}{1 \text{ revolution}} \cdot \frac{2\pi \text{ radians}}{1 \text{ revolution}}$$

$$= 80\pi \text{ ft/sec}$$

$$= 251.327 \text{ ft/sec}$$

radius = 10ft

$\frac{\Delta\theta}{\Delta t}$ change in angle
change in time

$\frac{\Delta d}{\Delta t}$ change in distance
change in time

- 3) A 45-rpm record has an angular speed (or velocity) of 45 revolutions per minute. ← given
Find the linear speed in inches per minute at the point where the needle is 1.5 inches from the record's center.

$$\begin{aligned} & 45 \frac{\text{rev}}{\text{min}} \cdot \frac{1.5 \text{ in}}{\text{radian}} \cdot \frac{2\pi \text{ radian}}{1 \text{ rev}} \\ &= 135\pi \text{ in/min} \\ &= 424.115 \text{ in/min} \end{aligned}$$

radius = 1.5 in

Final answer

$\frac{\Delta d}{\Delta t}$ change in distance / change in time



- 4) Ms. Gonzalez's truck has wheels 36" in diameter and she is driving along Austin Blvd outside Morton East. If her wheels are rotating at 320 rpm, is she speeding? (Note: There are 5280ft per mile.)

diameter = 36 inches
so, radius = 18 inches

$$\begin{aligned} & 320 \frac{\text{rev}}{\text{min}} \cdot \frac{18 \text{ in}}{1 \text{ radian}} \cdot \frac{2\pi \text{ radian}}{1 \text{ rev}} \\ &= 11520\pi \frac{\text{in}}{\text{min}} \\ &= 11520\pi \frac{\text{in}}{\text{min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{1 \text{ ft}}{12 \text{ in}} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}} \\ &= 34.271 \frac{\text{miles}}{\text{hr}} \end{aligned}$$

need miles/hr

Speed limit on Austin is 25 mph.
Ms. G was speeding.

Nautical Miles

A nautical mile is the length of 1 minute of an arc along the earth's equator.

$$1 \text{ statute mile} = \frac{10800}{3956\pi} \text{ nautical miles}$$

$$1 \text{ nautical mile} = \frac{3956\pi}{10800} \text{ statute miles}$$

Example

Mr. Januszyk flew from Boston to San Francisco a distance of 2698 statute miles. How many nautical miles is this equivalent to?

$$2698 \text{ statute miles} \left(\frac{\frac{10800}{3956\pi} \text{ nautical miles}}{1 \text{ statute miles}} \right)$$

2344.550 nautical miles