DATE: _____

Unit 5 (Chapter 4): Trigonometric Functions **Pre-Calculus**

4.4 Graphs of Sinusoidal Functions

Target 5C: Rigid and non-rigid transformations of sinusoids

Review of Prior Concepts

From the parent function $f(x) = x^2$, describe the transformation of $g(x) = (x-1)^2 + 3$ and give the domain and range of $g(x)$.

vertical shift up 3 units,
horizontal shift right 1 unit

Domain: $(-\infty, \infty)$

Range: $[3, \infty)$

More Practice

Transformations

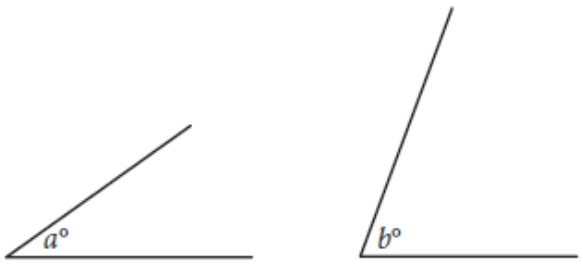
https://www.mathsisfun.com/sets/function-transformations.html
https://www.khanacademy.org/math/algebra2/manipulating-functions/stretching-functions/v/shifting-and-reflecting-functions
https://academics.utep.edu/Portals/1788/CALCULUS%20MATERIAL/1_7%20TRANSFORMATION%20OF%20FNS.pdf
https://www.youtube.com/watch?v=0a-AjP4UdnY
https://www.youtube.com/watch?v=3Q5Sy034fok

SAT Connection

Passport to Advanced Math

14. Use structure to isolate or identify a quantity of interest in an expression

Example:



Note: Figures not drawn to scale.

The angles shown above are acute and $\sin(a°) = \cos(b°)$. If $a = 4k - 22$ and $b = 6k - 13$, what is the value of k ?

A) 4.5
B) 5.5
C) 12.5
D) 21.5

Since $\sin(a) = \cos(b)$

$a = 90 - b$

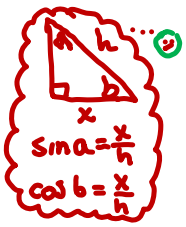
$4k - 22 = 90 - (6k - 13)$

$4k - 22 = 90 - 6k + 13$

$4k - 22 = -6k + 103$

$10k = 125$

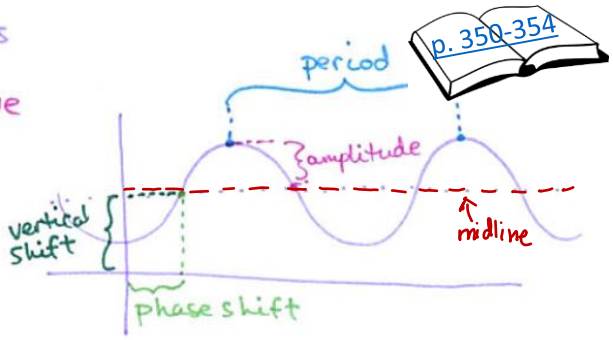
$k = 12.5$



Solution

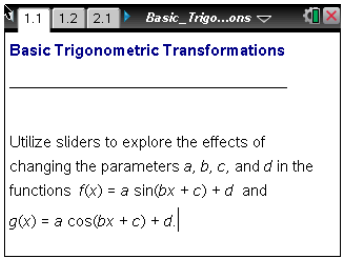
Vocabulary

- Sinusoidal Functions – sine and cosine functions
- Amplitude – $\frac{1}{2}$ height of the sine/cosine wave
- Period – one cycle of the sine wave
- Phase Shift – horizontal (left/right) shift
- Vertical Shift – up/down shift
- Midline – horizontal axis through the middle y-value



TI-Nspire Activity

Open the TI-Nspire document: *Basic_Trigonometric_Transformations.tns*



Move to page 1.2

1. Drag the sliders to change the values of a in the function $f(x) = a\sin(bx)$.

a) How does the value of a affect the shape of the graph?

vertical shrink or stretch

b) What happens to the graph if a is negative?

reflects over the x-axis

c) How does the value of b affect the shape of the graph?

horizontal shrink or stretch

Conclusion:

For $a \neq 0$ and $b \neq 0$, the graph of $f(x) = a\sin(bx)$ has an amplitude of $|a|$ and a period of $\frac{2\pi}{|b|}$.

Move to page 2.2

2. Drag the sliders to change the value of d in the function of $f(x) = \sin(x) + d$.

How does the value of d affect the shape of the graph?

vertical shift (up/down)

Conclusion:

The graph of $f(x) = \sin(x) + d$ has a vertical shift of d.

Move to page 3.2

3. Drag the sliders to change the value of c in the function of $f(x) = \sin(x + c)$.

How does the value of c affect the shape of the graph?

horizontal (left/right) shift

Conclusion:

The graph of $f(x) = \sin(x + c)$ has a phase shift of c.

c>0, shift left
c<0, shift right

Move to page 4.2

4. Drag the sliders to change the value of *a*, *b*, *c* and *d* in the function $f(x) = a\sin(bx + c) + d$.
Which of the four parameters have an impact on the phase shift of the graph?

c and b

$a\sin(b(x+\frac{c}{b}))+d$

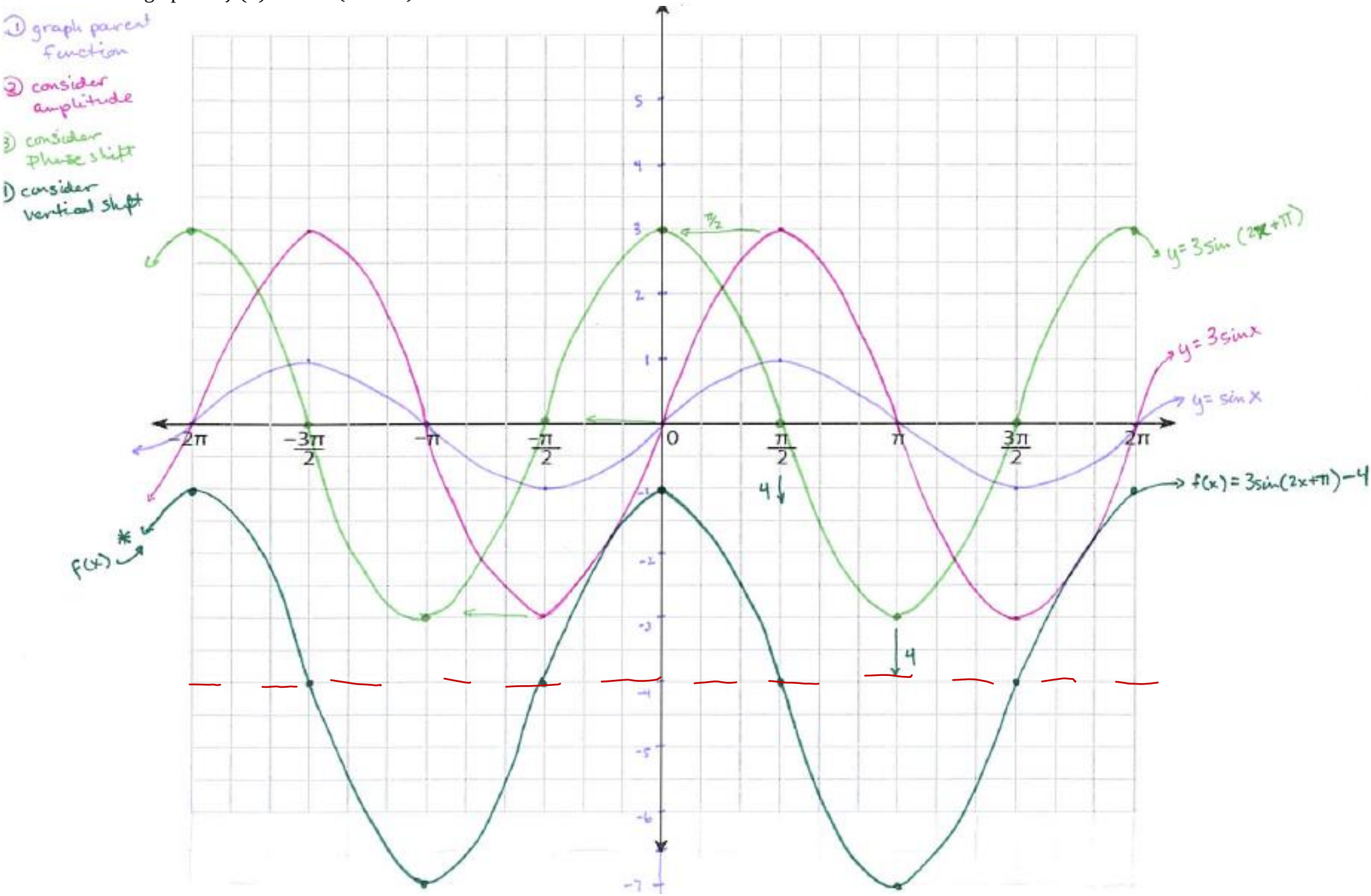
phase shift

Conclusion:
The graph of $f(x) = a\sin(bx + c) + d$ has a phase shift of $\frac{c}{b}$.

Apply Knowledge from Activity

Transformation	**General Form** $f(x) = a\sin(bx + c) + d$ OR $f(x) = a\cos(bx + c) + d$	**Example** $f(x) = 3\sin(2x + \pi) - 4$
Amplitude	$\lvert a\rvert$	3
Period	$\frac{2\pi}{\lvert b\rvert}$	$\frac{2\pi}{\lvert 2\rvert} = \frac{2\pi}{2} = \pi$
Phase Shift	$\frac{c}{b}$	$\frac{\pi}{2}$ (to the left)
Vertical Shift	d	4 (down)

Sketch the graph of: $f(x) = 3\sin(2x + \pi) - 4$



Unit 5 (Chapter 4): Trigonometric Functions

More Practice

Transformations of Sinusoidal Functions

https://www.khanacademy.org/math/trigonometry/trig-function-graphs
http://www.purplemath.com/modules/grphtrig.htm
http://www.algebralab.org/lessons/lesson.aspx?file=Trigonometry_TrigTransformations.xml
https://www.youtube.com/watch?v=iEbF1aa0Qps
https://www.youtube.com/watch?v=s_NI50p-pcg

Homework Assignment

p.357 #3,5,9,11,13,15,21,25

SAT Connection

Solution

Choice C is correct. Since the angles are acute and $\sin(a°) = \cos(b°)$, it follows from the complementary angle property of sines and cosines that $a + b = 90$. Substituting $4k - 22$ for a and $6k - 13$ for b gives $(4k - 22) + (6k - 13) = 90$, which simplifies to $10k - 35 = 90$. Therefore, $10k = 125$, and $k = 12.5$.

Choice A is incorrect and may be the result of mistakenly assuming that $a + b$ and making a sign error. Choices B and D are incorrect because they result in values for a and b such that $\sin(a°) \neq \cos(b°)$.