

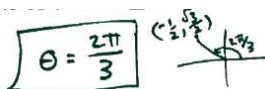
4.7 Inverse Trig Functions

Target 5D: Evaluate inverse and composite trigonometric functions and expressions using the unit circle
Review of Prior Concepts

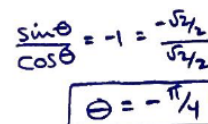
1) If $\sin \theta = \frac{\sqrt{2}}{2}$, find the value of θ for $-\frac{\pi}{2} \leq \theta < \frac{\pi}{2}$.



2) If $\cos \theta = -\frac{1}{2}$, find the value of θ for $0 \leq \theta < \pi$.



3) If $\tan \theta = -1$, find the value of θ for $-\frac{\pi}{2} \leq \theta < \frac{\pi}{2}$.



More Practice

Trig Values from Unit Circle

- <http://www.mathmistakes.info/facts/TrigFacts/learn/vals/sum.html>
- <http://www.kwarp.com/portfolio/trigspinner.html>
- <https://www.mathsisfun.com/geometry/unit-circle.html>
- <https://www.youtube.com/watch?v=LE6dmczMc68>
- <https://www.youtube.com/watch?v=RLjyGKWMSx0>

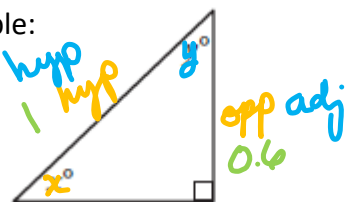


SAT Connection

Passport to Advanced Math

14. Use structure to isolate or identify a quantity of interest in an expression

Example:



$$\cos y = \frac{0.6}{1} = 0.6$$

In the triangle above, the sine of x° is 0.6. What is the cosine of y° ?

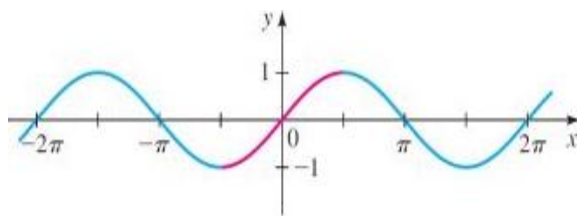
Solution

0	.	6
/	○	○
.	○	○
0	●	○
1	○	○
2	○	○
3	○	○
4	○	○
5	○	○
6	○	○
7	○	○
8	○	○
9	○	○

NOTE: You may start your answers in any column, space permitting. Columns you don't need to use should be left blank.

Inverse Sine Function

$y = \sin x$



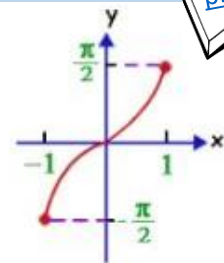
Restricted Domain: $[-\frac{\pi}{2}, \frac{\pi}{2}]$

Range: $[-1, 1]$

$y = \sin^{-1} x$

or

$y = \arcsin x$

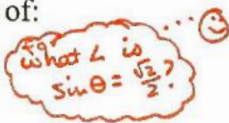


Domain: $[-1, 1]$

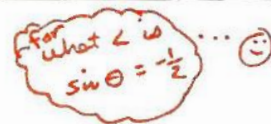
Range: $[-\frac{\pi}{2}, \frac{\pi}{2}]$

Example: Find the exact value of:

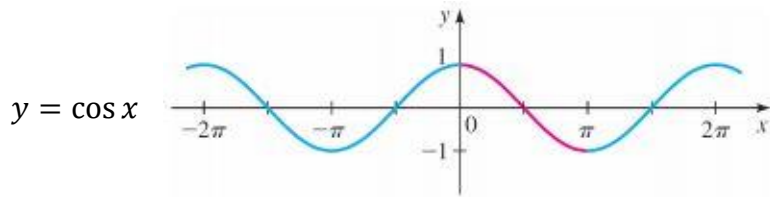
a) $\sin^{-1}(\frac{\sqrt{2}}{2}) = \frac{\pi}{4}$



b) $\arcsin(-\frac{1}{2}) = -\frac{\pi}{6}$



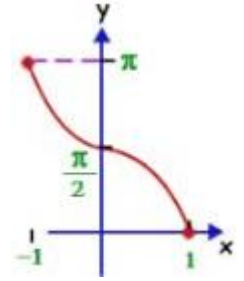
Inverse Cosine Function



$y = \cos^{-1} x$

or

$y = \arccos x$



Restricted Domain: $[0, \pi]$

Range: $[-1, 1]$

Domain: $[-1, 1]$

Range: $[0, \pi]$

Example: Find the exact value of:

a) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$
 $= \frac{\pi}{6}$

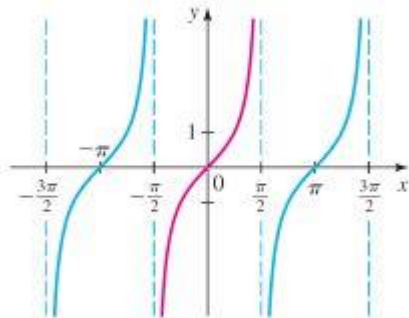
for what $\angle \theta$ is $\cos \theta = \frac{\sqrt{3}}{2}$?

b) $\arccos\left(-\frac{\sqrt{3}}{2}\right)$
 $= \frac{5\pi}{6}$

for what $\angle \theta$ is $\cos \theta = -\frac{\sqrt{3}}{2}$?

Inverse Tangent Function

$y = \tan x$



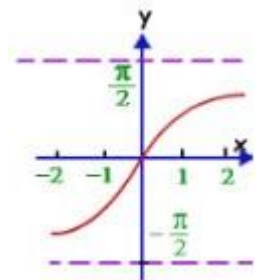
Restricted Domain: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Range: $(-\infty, \infty)$

$y = \tan^{-1} x$

or

$y = \arctan x$



Domain: $(-\infty, \infty)$

Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

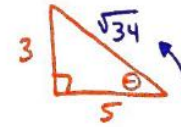
Composition of Inverse Trig Function

Composition of Inverse Trig Functions

- ① Make a Δ from inverse function
- ② Find missing side in Δ
- ③ Use Δ to get trig value of θ

Example:

Find the exact value of $\sin\left(\tan^{-1}\left(\frac{3}{5}\right)\right)$

① 

② $3^2 + 5^2 = c^2$
 $9 + 25 = c^2$
 $34 = c^2$
 $\sqrt{34} = c$

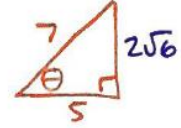
③ $\sin(\theta) = \frac{3}{\sqrt{34}}$

Handwritten notes: "for what \angle is $\tan \theta = \frac{3}{5}$?", "opp hyp", "adj hyp".

Example:

Evaluate without using a calculator.

a) $\tan\left(\arccos\left(\frac{5}{7}\right)\right)$

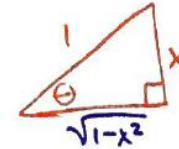
① 

② $5^2 + a^2 = 7^2$
 $25 + a^2 = 49$
 $a^2 = 24$
 $a = \sqrt{24}$
 $a = 2\sqrt{6}$

③ $\tan(\theta) = \frac{2\sqrt{6}}{5}$

Handwritten notes: "what \angle is $\cos \theta = \frac{5}{7}$ ", "opp adj".

b) $\cos(\sin^{-1} x)$

① 

② $a^2 + x^2 = 1^2$
 $a^2 + x^2 = 1$
 $a^2 = 1 - x^2$
 $a = \sqrt{1 - x^2}$

③ $\cos(\theta) = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$

Handwritten notes: "what \angle is $\sin \theta = \frac{x}{1}$ ", "adj hyp".

More Practice

Evaluating Inverse Trig Expressions

<http://tutorial.math.lamar.edu/Extras/AlgebraTrigReview/InverseTrig.aspx>

<http://www.themathpage.com/atrig/inversetrig.htm>

<https://www.youtube.com/watch?v=g9S4u8eQiw>

https://www.youtube.com/watch?v=pNgkK_MR6jM

Homework Assignment

p.385 #5,9,13,19,23-29odd,47,49,51

SAT Connection**Solution**

The correct answer is $.6$ or $\frac{3}{5}$. The angles marked x° and y° are acute angles in a right triangle. Thus, they are complementary angles. By the complementary angle relationship between sine and cosine, it follows that $\sin(x^\circ) = \cos(y^\circ)$. Therefore, the cosine of y° is $.6$. Either $.6$ or the equivalent fraction $\frac{3}{5}$ may be gridded as the correct answer.