4.8 Solving Problems with Trig

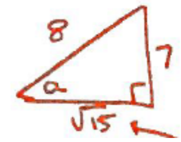
Target 5D: Evaluate inverse and composite trigonometric functions and expressions using the unit circle

Review of Prior Concepts

If $\sin a = \frac{7}{8}$, what is the value of $\cos a$?

$\cos a = \frac{adj}{hyp}$

$= \frac{\sqrt{15}}{8}$



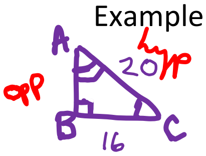
$x^2+7^2=8^2$
$x^2+49=64$
$x^2=15$
$x=\sqrt{15}$

More Practice

Trigonometry
https://www.khanacademy.org/math/trigonometry/trigonometry-right-triangles
http://www.mathsisfun.com/algebra/trigonometry.html
http://www.mathgoodies.com/lessons/vol2/circumference.html
https://www.youtube.com/watch?v=SqFQZWRALGc
https://www.youtube.com/watch?v=Jsiy4TxgIME

SAT Connection
Passport to Advanced Math
14. Use structure to isolate or identify a quantity of interest in an expression

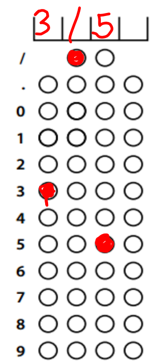
Example:



In triangle ABC, the measure of $\angle B$ is 90°, $BC = 16$, and $AC = 20$. Triangle DEF is similar to triangle ABC, where vertices D, E, and F correspond to vertices A, B, and C, respectively, and each side of triangle DEF is $\frac{1}{3}$ the length of the corresponding side of triangle ABC. What is the value of $\sin F$?

$(AB)^2+16^2=20^2$
$(AB)^2+256=400$
$(AB)^2=144$
$AB=12$

$\sin F = \sin C$
$\sin F = \frac{12}{20}$
$= \frac{3}{5}$

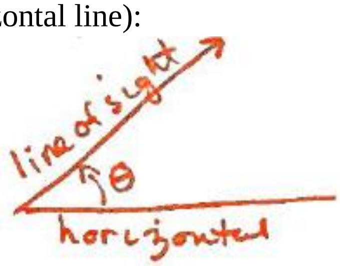


NOTE: You may start your answers in any column, space permitting. Columns you don't need to use should be left blank.

Solution

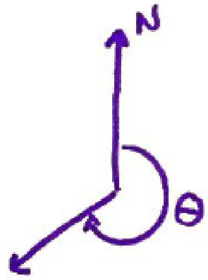
Terminology

Angle of elevation (measure with respect to a horizontal line):

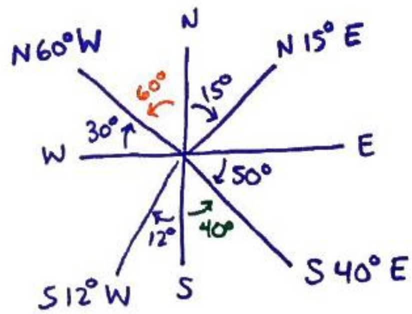


Angle of depression (measure with respect to a horizontal line):

Navigational angle (measure with respect to north, positive direction is clockwise):

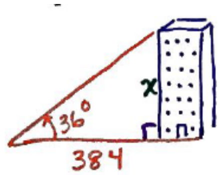


Surveying, bearing angle (the acute angle at which the direction varies to the east or west from the north-south line):



Examples

1) From a point 384 ft in a horizontal line from the base of a building, the angle of elevation to the top of the building is 36°. How tall is the building?



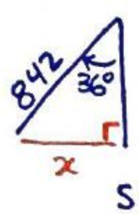
$\tan 36° = \frac{x}{384}$

$384 \tan 36° = x$

$278.992 = x$

The building is 278.992 ft tall

2) A certain piece of land is in the shape of a right triangle. The longest side is 842 meters and bears S 36° W. If one of the sides runs north-south, how long is the side that runs east-west?



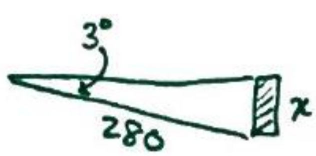
$\sin 36° = \frac{x}{842}$

$842 \sin 36° = x$

$494.915 = x$

The side that runs east-west is 494.915 meters

3) A piece of land slopes at an angle of 3° and runs for 280 ft in the direction of the slope. In order to level the land, a retaining wall is to be built at the lower end of the property so that fill-dirt can level the property. How high must the wall be?



$\sin 3° = \frac{x}{280}$

$280 \cdot \sin 3° = x$

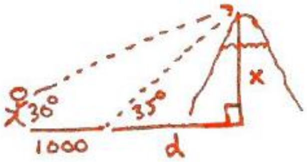
$14.654 = x$

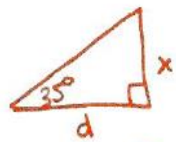
The wall must be 14.654 ft tall

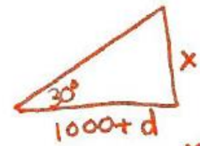
4) p. 394 #14

p.394

While hiking on a level path toward Colorado's Front Range, Otis Evans determines that the angle of elevation to the top of Long's Peak is 30°. Moving 1000 ft closer to the mountain, Otis determines the angle of elevation to be 35°. How much higher is the top of Long's Peak than Otis' elevation?







$d \cdot \tan 35° = \frac{x}{d} \cdot d$

$d \tan 35° = x$

$d = \frac{x}{\tan 35°}$

$\tan 30° = \frac{x}{1000+d}$

$\tan 30° = \frac{x}{1000 + \frac{x}{\tan 35°}}$

$\left(1000 + \frac{x}{\tan 35°}\right)\tan 30° = x$

$\tan 35° \cdot \left(1000 \tan 30° + \frac{x \cdot \tan 30°}{\tan 35°}\right) = x \cdot \tan 35°$

$1000 \tan 30° \tan 35° + x \tan 30° = x \tan 35°$

$1000 \tan 30° \tan 35° = x \tan 35° - x \tan 30°$

$1000 \tan 30° \tan 35° = x(\tan 35° - \tan 30°)$

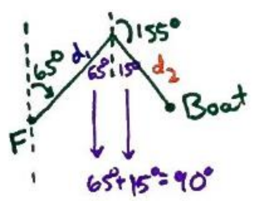
$\frac{1000 \tan 30° \tan 35°}{\tan 35° - \tan 30°} = x$

$\boxed{3290.526 \text{ ft}} = x$

5) p. 394 #18

p.395

The *Cerrito Lindo* travels at a speed of 40 knots from Fort Lauderdale on a course of 65° for 2 hours and then changes to a course of 155° for 4 hours. Determine the distance and the bearing from Fort Lauderdale to the boat.



knots = speed, hrs = time

distance = (rate)(time)
= (knots)(hrs)

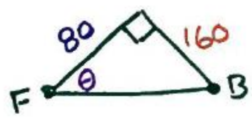
$d_1 = (40)(2)$
$= 80$ nautical miles

$d_2 = (40)(4)$
$= 160$ nautical miles

$FB^2 = 160^2 + 80^2$

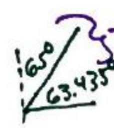
$FB = \sqrt{160^2 + 80^2}$

$= \boxed{178.885 \text{ nautical miles}}$



$\tan\theta = \frac{160}{80}$

$\theta = \tan^{-1}\left(\frac{160}{80}\right) = 63.435°$



$65° + 63.435°$

$\boxed{128.435° \text{ bearing}}$

More Practice

Trigonometric Ratios

http://www.themathpage.com/atrig/solve-right-triangles.htm

http://www.mathguide.com/lessons/RightTriTrig.html

https://www.youtube.com/watch?v=l5VbdqRjTXc

Homework Assignment

p.393 #3,6,10,13,15,

p.394 #16,17,23,25

SAT Connection

Solution

The correct answer is $\frac{3}{5}$ or .6. Triangle ABC is a right triangle with its right angle at B. Thus, $\overline{AC}$ is the hypotenuse of right triangle ABC, and $\overline{AB}$ and $\overline{BC}$ are the legs of right triangle ABC. By the Pythagorean theorem, $AB = \sqrt{20^2 - 16^2} = \sqrt{400 - 256} = \sqrt{144} = 12$. Since triangle DEF is similar to triangle ABC, with vertex F corresponding to vertex C, the measure of angle F equals the measure of angle C. Thus, $\sin F = \sin C$. From the side lengths of triangle ABC, $\sin C = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{12}{20} = \frac{3}{5}$. Therefore, $\sin F = \frac{3}{5}$. Either $\frac{3}{5}$ or its decimal equivalent, .6, may be gridded as the correct answer.