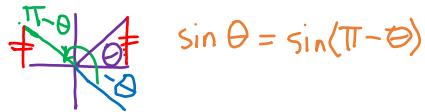


5.1 Fundamental Trig Identities

Target 6A: Verify, evaluate, and apply trigonometric identities and formulas

Review of Prior Concepts

If $\sin \theta = 0.57$, then $\sin(\pi - \theta) = ?$
 $\theta = \sin^{-1}(0.57)$ $\sin(\pi - 0.594) = 0.57$
 $\theta = 0.594$



Reciprocal Identities

$$\sin \theta = \frac{1}{\csc \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\tan \theta = \frac{1}{\cot \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

Quotient Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

Examples

Simplify.

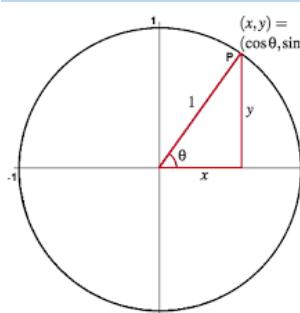
a) $\cot x \tan x$

$$= \frac{1}{\tan x} \cdot \tan x$$

$$= \boxed{1}$$

b) $\frac{\sin b}{\tan b} = \frac{\sin b}{\frac{\sin b}{\cos b}}$
 $= \sin b \cdot \frac{\cos b}{\sin b}$
 $= \boxed{\cos b}$

Pythagorean Identities



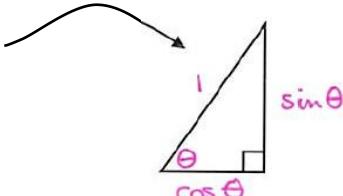
use, $\cos^2 \theta + \sin^2 \theta = 1$

$$\frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta}$$

$$1 + \left(\frac{\sin \theta}{\cos \theta}\right)^2 = \left(\frac{1}{\cos \theta}\right)^2$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$\tan^2 \theta = \sec^2 \theta - 1$
 $1 = \sec^2 \theta - \tan^2 \theta$



divide by $\cos^2 \theta$

$$\begin{aligned} a^2 + b^2 &= c^2 \\ (\cos \theta)^2 + (\sin \theta)^2 &= 1^2 \\ \cos^2 \theta + \sin^2 \theta &= 1 \end{aligned}$$

$\sin^2 \theta = 1 - \cos^2 \theta$
 $\cos^2 \theta = 1 - \sin^2 \theta$

use, $\cos^2 \theta + \sin^2 \theta = 1$ divide by $\sin^2 \theta$

$$\frac{\cos^2 \theta}{\sin^2 \theta} + \frac{\sin^2 \theta}{\sin^2 \theta} = \frac{1}{\sin^2 \theta}$$

$$\left(\frac{\cos \theta}{\sin \theta}\right)^2 + 1 = \left(\frac{1}{\sin \theta}\right)^2$$

$$\cot^2 \theta + 1 = \csc^2 \theta$$

$\cot^2 \theta = \csc^2 \theta - 1$
 $1 = \csc^2 \theta - \cot^2 \theta$

Unit 6 (Chapter 5): Analytic Trigonometry

Pre-Calculus

Examples

- a) Use Pythagorean Identities to find $\sec \theta$ and $\csc \theta$
if $\tan \theta = 3$ and $\cos \theta > 0$.

$$\begin{aligned} 1 + \tan^2 \theta &= \sec^2 \theta && \text{cos } \theta \text{ positive... so, sec } \theta \\ 1 + (3)^2 &= \sec^2 \theta && \text{also positive} \\ \sqrt{10} &= \sqrt{\sec^2 \theta} && \text{and if cos } \theta \text{ pos,} \\ \pm \sqrt{10} &= \sec \theta && \text{and tan } \theta \text{ pos,} \\ \pm \sqrt{10} &= \sec \theta && \text{then sin } \theta \text{ also} \\ &&& \text{positive} \\ &&& \text{:(} \end{aligned}$$

$$\begin{aligned} \tan \theta = 3 \rightarrow \cot \theta &= \frac{1}{3} \\ \cot^2 \theta + 1 &= \csc^2 \theta \\ (\frac{1}{3})^2 + 1 &= \csc^2 \theta \\ \frac{1}{9} + 1 &= \csc^2 \theta \\ \sqrt{\frac{10}{9}} &= \sqrt{\csc^2 \theta} \end{aligned}$$

$$\frac{\sqrt{10}}{3} = \csc \theta$$

b) Simplify $\frac{\sin^2 \alpha + \tan^2 \alpha + \cos^2 \alpha}{\sec \alpha}$

$$\begin{aligned} &= \frac{\sin^2 \alpha + \cos^2 \alpha + \tan^2 \alpha}{\sec \alpha} \\ &= \frac{1 + \tan^2 \alpha}{\sec \alpha} \\ &= \frac{\sec^2 \alpha}{\sec \alpha} \\ &= \boxed{\sec \alpha} \end{aligned}$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta \quad \cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta \quad \cot\left(\frac{\pi}{2} - \theta\right) = \tan \theta$$

$$\sec\left(\frac{\pi}{2} - \theta\right) = \csc \theta \quad \csc\left(\frac{\pi}{2} - \theta\right) = \sec \theta$$

Odd-Even Identities

$$\sin(-x) = -\sin x \quad \csc(-x) = -\csc x$$

$$\cos(-x) = \cos x \quad \sec(-x) = \sec x$$

$$\tan(-x) = -\tan x \quad \cot(-x) = -\cot x$$

Examples

- a) If $\tan\left(\frac{\pi}{2} - \theta\right) = -5.326$, find $\cot \theta$.

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta$$

$$\therefore, \boxed{\cot \theta = -5.326}$$

- b) Simplify $\sec(-x) \cos(-x)$

$$\begin{aligned} \sec(-x) \cos(-x) &= \sec x \cdot \cos x \\ &= \frac{1}{\cos x} \cdot \cos x \\ &= \boxed{1} \end{aligned}$$

$$\begin{aligned} \sec(-x) &= \sec x \\ \cos(-x) &= \cos x \end{aligned}$$



More Practice**Fundamental Trig Identities**

<http://www.intmath.com/analytic-trigonometry/1-trigonometric-identities.php>

<http://www.mathguide.com/lessons2/TrigExpress.html>

<http://www.purplemath.com/modules/proving.htm>

<https://www.youtube.com/watch?v=CsfHFZL345M>

<https://www.youtube.com/watch?v=I4mcja8abDc>

Homework Assignment

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