

## 5.1 Fundamental Trig Identities

Target 6A: Verify, evaluate, and apply trigonometric identities and formulas

### Review of Prior Concepts

Simplify.

$$\text{a) } \frac{3x+6}{x^2+5x+6} = \frac{3(x+2)}{(x+3)(x+2)} = \frac{3}{x+3}$$

factor + reduce

$$\text{b) } \frac{x}{2} + \frac{2}{x} - \frac{x}{4} = \frac{4 \cdot x}{2x} + \frac{4 \cdot 2}{4 \cdot x} - \frac{x \cdot x}{4 \cdot x} = \frac{2x^2 + 8 - x^2}{4x} = \frac{x^2 + 8}{4x}$$

common denominator

### More Practice

#### Simplifying Algebraic Expressions

<http://www.purplemath.com/modules/rtnldefs3.htm>

<https://www.khanacademy.org/math/algebra2/rational-expressions-equations-and-functions/simplify-rational-expressions/v/simplifying-rational-expressions-1>

<http://www.mathwarehouse.com/algebra/rational-expression/how-to-simplify-rational-expressions.php>

<https://www.youtube.com/watch?v=5xgRFbkQYgE>

<https://www.youtube.com/watch?v=NjBvycGQX48>

Simplify the following trig expressions. Check your answer in your graphing calculator.

As a class,

$$\begin{aligned} \text{1) } \tan^2 x (\csc^2 x - 1) &= \tan^2 x (\cancel{\csc^2 x} (1 + \cot^2 x) - 1) \\ &= \tan^2 x (\cot^2 x) \\ &= \tan^2 x \left( \frac{1}{\tan^2 x} \right) \\ &= \boxed{1} \end{aligned}$$

$\csc^2 x = 1 + \cot^2 x$

$$\begin{aligned} \text{2) } \frac{\tan x}{\csc^2 x} + \frac{\tan x}{\sec^2 x} &= \frac{\tan x}{\frac{1}{\sin^2 x}} + \frac{\tan x}{\frac{1}{\cos^2 x}} \\ &= \tan x \cdot \sin^2 x + \tan x \cdot \cos^2 x \\ &= \tan x (\sin^2 x + \cos^2 x) \\ &= \tan x (1) \\ &= \boxed{\tan x} \end{aligned}$$

$\csc^2 x = \frac{\sin^2 x}{\cos^2 x}$   
 $\sec^2 x = \frac{1}{\cos^2 x}$   
 $\sin^2 x + \cos^2 x = 1$

Now, you try:

$$\begin{aligned} \text{1) } \frac{\cos^2 x}{1 - \cos^2 x} &= \frac{\cos^2 x}{\sin^2 x + \cos^2 x - \cos^2 x} \\ &= \frac{\cos^2 x}{\sin^2 x} \\ &= \boxed{\cot^2 x} \end{aligned}$$

$\sin^2 x + \cos^2 x = 1$

$$\begin{aligned} \text{2) } \frac{\sin x}{\cot^2 x} + \frac{\sin x}{\cos^2 x} &= \frac{\sin x}{\frac{1}{\tan^2 x}} + \frac{\sin x}{\frac{1}{\sec^2 x}} \\ &= \sin x \cdot \tan^2 x + \sin x \cdot \sec^2 x \\ &= \sin x (\tan^2 x + \sec^2 x) \\ &= \sin x (1 + \tan^2 x + \sec^2 x) \\ &= \sin x (2 \tan^2 x + 1) \\ &= \boxed{\sin x (2 \tan^2 x + 1)} \end{aligned}$$

$\cot^2 x = \frac{1}{\tan^2 x}$   
 $\sec^2 x = \frac{1}{\cos^2 x}$   
 $\tan^2 x + 1 = \sec^2 x$

## Unit 6 (Chapter 5): Analytic Trigonometry

## Pre-Calculus

**Factor the expression as a single trigonometric function**

As a class,

$$3) \cos x - 2 \sin^2 x + 1$$

$$= \cos x - 2(1 - \cos^2 x) + 1$$

$$= \cos x - 2 + 2\cos^2 x + 1$$

$$= 2\cos^2 x + \cos x - 1 \quad \text{let } u = \cos x$$

$$2u^2 + u - 1$$

$$(2u - 1)(u + 1)$$

$$= \boxed{(2\cos x - 1)(\cos x + 1)}$$

Now, you try:

$$3) \sin^2 x + \frac{2}{\csc x} + 1$$

$$= \sin^2 x + 2 \sin x + 1$$

$$\text{let } u = \sin x$$

$$u^2 + 2u + 1$$

$$(u+1)(u+1)$$

$$= (\sin x + 1)(\sin x + 1)$$

$$= \boxed{(\sin x + 1)^2}$$

### RALLY COACH

Simplify each expression. Check your answer graphically. Take turns doing problems.

Name:	Name:
$\frac{1 - \cos^2 \theta}{\sin \theta} = \frac{\sin^2 \theta}{\sin \theta}$ $= \boxed{\sin \theta}$	$\frac{\sin^2 \theta \cot^2 \theta}{1 - \sin^2 \theta} = \frac{\sin^2 \theta \cdot \frac{\cos^2 \theta}{\sin^2 \theta}}{\frac{1 - \sin^2 \theta}{\cos^2 \theta}}$ $= \frac{\cos^2 \theta}{\cos^2 \theta}$ $= \boxed{1}$
$\cos x \csc x (\sec^2 x - 1)$ $= \cos x \cdot \frac{1}{\sin x} (\tan^2 x)$ $= \frac{\cos x}{\sin x} \cdot \left( \frac{\sin^2 x}{\cos^2 x} \right)$ $= \frac{\sin x}{\cos x}$ $= \boxed{\tan x}$	$\frac{\sec^2 x - 1}{\tan x} = \frac{\tan^2 x}{\tan x}$ $= \boxed{\tan x}$

$$\begin{aligned}\frac{\tan \beta + \cot \beta}{\cot \beta} &= \frac{\tan \beta}{\cot \beta} + \frac{\cot \beta}{\cot \beta} \\&= \frac{\tan \beta}{1} + 1 \\&= \tan \beta \cdot \tan \beta + 1 \\&= \tan^2 \beta + 1 \\&= \boxed{\sec^2 \beta}\end{aligned}$$

$$\begin{aligned}&\cos \beta (\sec \beta - \cos \beta) \\&= \cos \beta \sec \beta - \cos^2 \beta \\&= \cos \beta \cdot \frac{1}{\cos \beta} - \cos^2 \beta \\&= 1 - \cos^2 \beta \\&= \boxed{\sin^2 \beta}\end{aligned}$$

$$\begin{aligned}\frac{\tan \alpha - \tan \alpha \sin^2 \alpha}{2 \sin \alpha \cos \alpha} &= \frac{\tan \alpha (1 - \sin^2 \alpha)}{2 \sin \alpha \cos \alpha} \\&= \frac{\tan \alpha (\cos^2 \alpha)}{2 \sin \alpha \cos \alpha} \\&= \frac{\tan \alpha \cdot \cos \alpha}{2 \sin \alpha} \\&= \frac{\sin \alpha}{2 \sin \alpha} \cdot \frac{\cos \alpha}{\cos \alpha} \quad \text{Simplify} \\&= \frac{1}{2}\end{aligned}$$

$$\begin{aligned}\frac{\sec \alpha - \cos \alpha}{2 \tan \alpha \sin \alpha} &= \frac{\frac{1}{\cos \alpha} - \cos \alpha}{2 \frac{\sin \alpha}{\cos \alpha} \cdot \sin \alpha} \\&= \frac{\frac{1 - \cos^2 \alpha}{\cos \alpha}}{2 \frac{\sin^2 \alpha}{\cos \alpha}} \\&= \frac{\frac{\sin^2 \alpha}{\cos \alpha}}{2 \frac{\sin^2 \alpha}{\cos \alpha}} \quad \text{Simplify} \\&= \frac{\sin^2 \alpha}{\cos \alpha} \cdot \frac{\cos \alpha}{2 \sin^2 \alpha} \\&= \boxed{\frac{1}{2}}\end{aligned}$$

**More Practice****Simplifying Trig Expressions**

- <http://www.intmath.com/analytic-trigonometry/1-trigonometric-identities.php>
- <http://www.mathguide.com/lessons2/TrigExpress.html>
- <http://www.purplemath.com/modules/proving.htm>
- <https://www.youtube.com/watch?v=CsfHFZL345M>
- <https://www.youtube.com/watch?v=l4mcja8abDc>

**Homework Assignment**

p.410 #25,29,33,37,41,45,49