DATE: _____

Unit 6 (Chapter 5): Analytic Trigonometry **Pre-Calculus**

5.3 Fundamental Trig Identities

Target 6B: Prove trigonometric identities

Sum and Difference Identities

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

Examples

Find the exact value.

1) $\sin\left(\frac{5\pi}{12}\right)$

$= \sin\left(\frac{3\pi}{12} + \frac{2\pi}{12}\right)$

$= \sin\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$

$\alpha = \pi/4$, $\beta = \pi/6$ ...

$= \sin\frac{\pi}{4}\cos\frac{\pi}{6} + \cos\frac{\pi}{4}\sin\frac{\pi}{6}$

$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$

$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} \Rightarrow \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

2) $\cos\left(\frac{5\pi}{12}\right)$

$= \cos\left(\frac{3\pi}{12} + \frac{2\pi}{12}\right)$

$= \cos\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$

$\alpha = \pi/4$, $\beta = \pi/6$ ...

$= \cos\frac{\pi}{4}\cos\frac{\pi}{6} - \sin\frac{\pi}{4}\sin\frac{\pi}{6}$

$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \Rightarrow \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

3) $\cos\left(\frac{11\pi}{12}\right)$

$= \cos\left(\frac{14\pi}{12} - \frac{3\pi}{12}\right)$

$= \cos\left(\frac{7\pi}{6} - \frac{\pi}{4}\right)$

$= \cos\frac{7\pi}{6}\cos\frac{\pi}{4} + \sin\frac{7\pi}{6}\sin\frac{\pi}{4}$

$= -\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + -\frac{1}{2}\cdot\frac{\sqrt{2}}{2}$

$= \frac{-\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= \boxed{\frac{-\sqrt{6}-\sqrt{2}}{4}}$ or $\boxed{-\frac{\sqrt{6}+\sqrt{2}}{4}}$

OR

$\cos\left(\frac{8\pi}{12} + \frac{3\pi}{12}\right)$

$= \cos\left(\frac{2\pi}{3} + \frac{\pi}{4}\right)$

$= \cos\frac{2\pi}{3}\cos\frac{\pi}{4} - \sin\frac{2\pi}{3}\sin\frac{\pi}{4}$

$= -\frac{1}{2}\cdot\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$

$= \frac{-\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$

$= \boxed{\frac{-\sqrt{6}-\sqrt{2}}{4}}$ or $\boxed{-\frac{\sqrt{6}+\sqrt{2}}{4}}$

What about Tangent?

$$\tan(\alpha \pm \beta) = \frac{\sin(\alpha \pm \beta)}{\cos(\alpha \pm \beta)}$$

$$\boxed{\tan(\alpha \pm \beta) = \frac{\sin\alpha\cos\beta \pm \cos\alpha\sin\beta}{\cos\alpha\cos\beta \mp \sin\alpha\sin\beta}}$$

OR

$$= \frac{\dfrac{\sin\alpha\cancel{\cos\beta}}{\cos\alpha\cancel{\cos\beta}} \pm \dfrac{\cancel{\cos\alpha}\sin\beta}{\cancel{\cos\alpha}\cos\beta}}{\dfrac{\cancel{\cos\alpha}\cancel{\cos\beta}}{\cancel{\cos\alpha}\cancel{\cos\beta}} \mp \dfrac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}$$

* divide each term by $\cos\alpha\cos\beta$

* and reduce

OR

$$\boxed{\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}}$$

Examples

Find the exact value.

4) $\tan(15°) = \tan(45° - 30°)$

$$= \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°}$$

$$= \frac{\dfrac{\sqrt{2}/2}{\sqrt{2}/2} - \dfrac{1/2}{\sqrt{3}/2}}{1 + \left(\dfrac{\sqrt{2}/2}{\sqrt{2}/2}\right)\left(\dfrac{1/2}{\sqrt{3}/2}\right)}$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}$$

$$= \frac{\left(1 - \frac{1}{\sqrt{3}}\right)\sqrt{3}}{\left(1 + \frac{1}{\sqrt{3}}\right)\sqrt{3}}$$

$$= \boxed{\frac{\sqrt{3} - 1}{\sqrt{3} + 1}}$$

5) $\tan\left(-\frac{\pi}{12}\right) = \tan\left(\frac{3\pi}{12} - \frac{4\pi}{12}\right)$

$$= \tan\left(\frac{\pi}{4} - \frac{\pi}{3}\right)$$

$$= \frac{\sin(\pi/4 - \pi/3)}{\cos(\pi/4 - \pi/3)}$$

$$= \frac{\sin\pi/4\cos\pi/3 - \cos\pi/4\sin\pi/3}{\cos\pi/4\cos\pi/3 + \sin\pi/4\sin\pi/3}$$

$$= \frac{\sqrt{2}/2 \cdot 1/2 - \sqrt{2}/2 \cdot \sqrt{3}/2}{\sqrt{2}/2 \cdot 1/2 + \sqrt{2}/2 \cdot \sqrt{3}/2}$$

$$= \frac{\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}}{\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}}$$

$$= \frac{\left(\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}\right)4}{\left(\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}\right)4}$$

$$= \frac{\sqrt{2} - \sqrt{6}}{\sqrt{2} + \sqrt{6}}$$

ok answer, but....

$$= \frac{\sqrt{2}(1 - \sqrt{3})}{\sqrt{2}(1 + \sqrt{3})}$$

$$= \boxed{\frac{1 - \sqrt{3}}{1 + \sqrt{3}}}$$

better answer

More Practice

Proof of Sum & Difference Identities

https://www.youtube.com/watch?v=nt0Nfz5Lc0A
https://www.youtube.com/watch?v=Jo2PhYS8vYE
http://www.themathpage.com/atrig/sum-proof.htm

Using Sum & Difference Identities

http://www.intmath.com/analytic-trigonometry/2-sum-difference-angles.php
http://www.onlinemathlearning.com/sum-identities.html
http://www.purplemath.com/modules/ideneval.htm
http://www.algebralab.org/lessons/lesson.aspx?file=Trigonometry_TrigSumDifference.xml
https://www.youtube.com/watch?v=ZhvvkCa_60w
https://www.youtube.com/watch?v=0ZFxY0uMJy0
https://www.youtube.com/watch?v=yklLtxBOb4s
https://www.youtube.com/watch?v=KuszlL_CJLU

Homework Assignment

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